Automatic Robust Linear Receiver for Multi-Access Space-Time Block Coded MIMO Systems

Jun Yang, Xiaochuan Ma, Chaohuan Hou, *Fellow, IEEE*, Yicong Liu, and Zheng Yao

***Abstract*—In this letter, we develop a fully automatic robust linear receiver technique for joint space-time decoding and interference rejection in multi-access MIMO systems that use orthogonal space-time block codes and erroneous channel state information (CSI). The proposed receiver does not need any *a priori* knowledge of channel estimation errors and has a simple closed form. Numerical examples show that our method usually gives good performance in case of non-perfect CSI and/or low sample sizes when compared with other tested linear receivers.**

***Index Terms*—Multi-access MIMO communications, orthogonal space-time block codes, robust linear receiver.**

I. Introduction

ORTHOGONAL space-time block codes (OSTBCs) [1], [2] represent an attractive class of space-time coding techniques since they enjoy full diversity and low decoding complexity. In the point-to-point MIMO communication case, maximum likelihood (ML) detection can be realized with a simple linear filter which can be interpreted as a space-time matched filter (MF) [3]. However, in the multi-access MIMO case, the ML receiver has much more complicated structure and prohibitively high complexity. Meanwhile, the MF receiver degrades severely because it does not take into account the structure of the multi-access interference (MAI). Therefore, efficient suboptimal receivers are needed. In last few years, several linear receiver techniques have been developed, such as [4]–[7]. A common shortcoming of these methods is that they use the assumption that the exact channel state information (CSI) is available at the receiver. However, this condition can be violated in practice because of channel estimation errors caused by, for example, MAI and/or outdated training. To cope with this, robust methods based on worst-case optimization [8], probability-constrained optimization [9] and adaptive projected subgradient method [10] have been proposed recently. However, the performance of the aforementioned methods depends on the choice of user parameters which are related to either the knowledge of CSI mismatches (see [8], [9]) or a properly chosen constant (see [10]). Unfortunately, the parameters may be difficult to estimate in practice.

In this letter, we derive a parameter-free robust method based on the minimum variance (MV) linear receiver in [7] with generalized loading, which can be seen as an extension to the well-known diagonal loading approach and has been successfully used for developing robust techniques for adaptive beamforming [11]. The proposed receiver has a couple of advantages compared with existing robust methods. One is that the proposed method is fully automatic, which means it does not need any *a priori* knowledge for choosing user parameter, such as the variance of errors on CSI used in [8], [9]. The other is that the computational complexity of the proposed method is about the same as the MV receiver, which may be much lower than those of other existing robust methods.

II. Problem Formulation

Consider an uplink multi-access MIMO communication system with M receiving antennas. The transmitters are assumed to have the same number of transmitting antennas which is denoted by N and to encode the information-bearing symbols using the same OSTBC.[1] Then, the received signal can be formulated by

$$\boldsymbol{Y}=\sum_{p=1}^{P}\boldsymbol{X}_p\boldsymbol{H}_p+\boldsymbol{F} \tag{1}$$

where $\boldsymbol{Y}$ is the matrix of received signals, $\boldsymbol{X}_p$ is the matrix of signals transmitted by the pth transmitter, $\boldsymbol{H}_p$ is the $N\times M$ complex channel matrix between the pth transmitter and the receiver, $\boldsymbol{F}$ is the matrix of noise, and P is the number of transmitters. Let $\boldsymbol{s}\triangleq(s_1,s_2,\ldots,s_K)^{\mathrm{T}}$ be the vector of K symbols transmitted by the user where $(\cdot)^{\mathrm{T}}$ denotes the transpose operator. The matrix $\boldsymbol{X}(\boldsymbol{s})$ is the $T\times N$ OSTBC, which can be written as [7]

$$\boldsymbol{X}(\boldsymbol{s})=\sum_{k=1}^{K}(\boldsymbol{C}_k\mathrm{Re}\{s_k\}+\boldsymbol{D}_k\mathrm{Im}\{s_k\}) \tag{2}$$

where $\boldsymbol{C}_k\triangleq\boldsymbol{X}(\tilde{\boldsymbol{e}}_k)$, $\boldsymbol{D}_k\triangleq\boldsymbol{X}(j\tilde{\boldsymbol{e}}_k)$, $j=\sqrt{-1}$, and $\tilde{\boldsymbol{e}}_k$ is the $K\times 1$ vector having one in the kth position and zeros elsewhere. By defining the "underline" operator for any matrix $\boldsymbol{M}$ as [7]

$$\underline{\boldsymbol{M}}\triangleq\begin{pmatrix}\mathrm{vec}\{\mathrm{Re}(\boldsymbol{M})\}\\ \mathrm{vec}\{\mathrm{Im}(\boldsymbol{M})\}\end{pmatrix} \tag{3}$$

where $\mathrm{vec}\{\cdot\}$ is the vectorization operator stacking all columns of a matrix on top of each other, (1) can be rewritten as [7]

$$\underline{\boldsymbol{Y}}=\sum_{p=1}^{P}\boldsymbol{A}_p\underline{\boldsymbol{s}_p}+\underline{\boldsymbol{F}} \tag{4}$$

[1]These assumptions are only needed for notational simplicity and can be relaxed [7].

Manuscript received February 19, 2009; revised April 26, 2009. First published May 12, 2009; current version published June 09, 2009. The associate editor coordinaintg the review of this manuscript and approving it for publication was Prof. Xiaoli Ma.

J. Yang is with Institute of Acoustics and the Graduate University, Chinese Academy of Sciences (CAS), Beijing 100190, China (e-mail: yangjun@mail.ioa.ac.cn).

X. Ma and C. Hou are with Institute of Acoustics, Chinese Academy of Sciences, Beijing 100190, China (e-mail: maxc@mail.ioa.ac.cn, hch@mail.ioa.ac.cn).

Y. Liu is with State Key Laboratory of Information Security, Chinese Academy of Sciences, Beijing 100049, China (e-mail: ycliu@is.ac.cn).

Z. Yao is with Department of Electronic Engineering, Tsinghua University, Beijing 100084, China (e-mail: yaozheng@ieee.org).

Digital Object Identifier 10.1109/LSP.2009.2022791

where $\boldsymbol{s}_p$ is a $K \times 1$ vector of the pth trasmitter's information-bearing symbols and the matrix $\boldsymbol{A}_p$ can be expressedas

$$\boldsymbol{A}_p = (\underline{\boldsymbol{C}_1\boldsymbol{H}_p}, \ldots, \underline{\boldsymbol{C}_K\boldsymbol{H}_p}, \underline{\boldsymbol{D}_1\boldsymbol{H}_p}, \ldots, \underline{\boldsymbol{D}_K\boldsymbol{H}_p}). \tag{5}$$

An important property of this matrix is that its columns have the same norms and are orthogonal to each other:

$$\boldsymbol{A}_p^{\mathrm{T}} \cdot \boldsymbol{A}_p = \|\boldsymbol{H}_p\|_F^2 \boldsymbol{I}_{2K} \tag{6}$$

where $\|\cdot\|_F$ denotes the Frobenius norm. Without loss of generality, we assume the first transmitter is the transmitter-of-interest. Then, the output vector of a linear receiver can be formulated by [7]

$$\underline{\hat{\boldsymbol{s}}_1} = \boldsymbol{W}^{\mathrm{T}}\underline{\boldsymbol{Y}} \tag{7}$$

where $\boldsymbol{W}$ is the $2MT \times 2K$ real matrix of the receiver coefficients. Based on the formulation of (7), we can interpret the MF receiver which becomes highly nonoptimal in the multi-access MIMO case with the following coefficient matrix:

$$\boldsymbol{W}_{\mathrm{MF}} = \boldsymbol{A}_1/\|\boldsymbol{H}_1\|_F^2. \tag{8}$$

The MV linear receiver which is able to reject both MAI and self-interference can be formulated as [7]

$$\min_{\boldsymbol{W}} \mathrm{tr}\{\boldsymbol{W}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{W}\} \quad \text{subject to } \boldsymbol{A}_1^{\mathrm{T}} \cdot \boldsymbol{W} = \boldsymbol{I}_{2K} \tag{9}$$

where $\mathrm{tr}\{\cdot\}$ is the trace operator, $\boldsymbol{R} = (1/J)\sum_{i=1}^{J} \underline{\boldsymbol{Y}_i} \cdot \underline{\boldsymbol{Y}_i^{\mathrm{T}}}$ is the sample estimate of the $2MT \times 2MT$ full-rank covariance matrix and J is the sample size. The solution to (9) can be written as

$$\boldsymbol{W}_{\mathrm{MV}} = \boldsymbol{R}^{-1}\boldsymbol{A}_1\left(\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{A}_1\right)^{-1}. \tag{10}$$

In the case of imperfect CSI and low sample size, it was suggested in [7] to apply fixed diagonal loading (DL) to improve the performance of the MV receiver (10).

III. Minimized Mean-Squared Error (MSE) Shrinkage Estimator

A. Definition of Shrinkage Estimator

The coefficient matrix $\boldsymbol{W}$ in (9) can be reparameterized by a new parameter matrix $\boldsymbol{B} \in \mathbb{C}^{(2MT-2K)\times 2K}$ according to

$$\boldsymbol{W} = \boldsymbol{A}_1/\|\boldsymbol{H}_1\|_F^2 - \boldsymbol{Q}\boldsymbol{B} \tag{11}$$

where $\boldsymbol{Q} \in \mathbb{C}^{2MT\times(2MT-2K)}$ is a semi-unitary matrix which spans the entire space together with $\boldsymbol{A}_1$ so that $\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{A}_1 = \boldsymbol{0}$ and $\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{Q} = \boldsymbol{I}_{2MT-2K}$. The structure in (11) shows the relation between the MF receiver and the MV receiver. Note that the first term $\boldsymbol{A}_1/\|\boldsymbol{H}_1\|_F^2$ in (11) is the MF receiver when there is no MAI. In the presence of MAI, the MV receiver subtract out the interference which is assumed orthogonal to $\boldsymbol{A}_1$. Hence the linear solution for interference suppression will be of the form $\boldsymbol{Q}\boldsymbol{B}$. In this way, (9) can be rewritten as

$$\begin{aligned}
&\min_{\boldsymbol{B}} \mathrm{tr}\left\{\left(\frac{\boldsymbol{A}_1}{\|\boldsymbol{H}_1\|_F^2} - \boldsymbol{Q}\boldsymbol{B}\right)^{\mathrm{T}} \boldsymbol{R}\left(\frac{\boldsymbol{A}_1}{\|\boldsymbol{H}_1\|_F^2} - \boldsymbol{Q}\boldsymbol{B}\right)\right\} \\
&= \min_{\boldsymbol{B}} \mathrm{tr}\left\{\left(\boldsymbol{R}^{\frac{1}{2}}\frac{\boldsymbol{A}_1}{\|\boldsymbol{H}_1\|_F^2} - \boldsymbol{R}^{\frac{1}{2}}\boldsymbol{Q}\boldsymbol{B}\right)^{\mathrm{T}}\right. \\
&\qquad \left. \times \left(\boldsymbol{R}^{\frac{1}{2}}\frac{\boldsymbol{A}_1}{\|\boldsymbol{H}_1\|_F^2} - \boldsymbol{R}^{\frac{1}{2}}\boldsymbol{Q}\boldsymbol{B}\right)\right\} \\
&= \min_{\boldsymbol{B}} \left\|\boldsymbol{R}^{\frac{1}{2}}\frac{\boldsymbol{A}_1}{\|\boldsymbol{H}_1\|_F^2} - \boldsymbol{R}^{\frac{1}{2}}\boldsymbol{Q}\boldsymbol{B}\right\|_F^2 \triangleq \sigma^2
\end{aligned} \tag{12}$$

with $\boldsymbol{R}^{1/2}$ to be the positive definite square root of $\boldsymbol{R}$. The solution can be formulated by

$$\boldsymbol{B}_{\mathrm{MV}} = (\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q})^{-1}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{A}_1/\|\boldsymbol{H}_1\|_F^2. \tag{13}$$

We also note that the solution of (12) is equivalent to the least squares (LS) solution of the linear regression problem

$$\left(\boldsymbol{R}^{\frac{1}{2}}\frac{\boldsymbol{A}_1}{\|\boldsymbol{H}_1\|_F^2}\right) = \left(\boldsymbol{R}^{\frac{1}{2}}\boldsymbol{Q}\right)\boldsymbol{B} + \boldsymbol{E}, \quad \boldsymbol{E} \sim \mathcal{N}(\boldsymbol{0}, \sigma^2\boldsymbol{I}) \tag{14}$$

where $\boldsymbol{E} \in \mathbb{C}^{2MT\times 2K}$ is the residual term. Hence we can compute the mean-squared error (MSE) of the $\boldsymbol{B}_{\mathrm{MV}}$ in the linear regression framework as

$$\begin{aligned}
\mathrm{MSE}(\boldsymbol{B}_{\mathrm{MV}}) &\triangleq \mathrm{E}\|\boldsymbol{B}_{\mathrm{MV}} - \boldsymbol{B}\|_F^2 \\
&= \mathrm{E}\left(\mathrm{tr}\left\{(\boldsymbol{B}_{\mathrm{MV}} - \boldsymbol{B})^{\mathrm{T}}(\boldsymbol{B}_{\mathrm{MV}} - \boldsymbol{B})\right\}\right) \\
&= \sigma^2 \mathrm{tr}\left\{\boldsymbol{R}^{\frac{1}{2}}\boldsymbol{Q}(\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q})^{-2}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}^{\frac{1}{2}}\right\} \\
&\quad + \mathrm{E}(\boldsymbol{E}^{\mathrm{T}})\boldsymbol{R}^{\frac{1}{2}}\boldsymbol{Q}(\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q})^{-2}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}^{\frac{1}{2}}\mathrm{E}(\boldsymbol{E}) \\
&= \sigma^2 \mathrm{tr}\left\{(\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q})^{-2}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q}\right\} \\
&= \sigma^2 \mathrm{tr}(\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q})^{-1}
\end{aligned} \tag{15}$$

in which $\mathrm{E}(\cdot)$ is the expectation operator. From (15), we note that the matrix $\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q}$ is usually ill-conditioned because of the existing of MAI in the multi-access case. Thus, the MSE of $\boldsymbol{B}_{\mathrm{MV}}$ is always high, and this will lead to the sensitivity of the MV receiver to CSI mismatches and errors on sample covariance matrix. Suppose that $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_{2MT-2K}$ are the eigenvalues of $\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q}$. Let the singular value decomposition (SVD) of $\boldsymbol{R}^{1/2}\boldsymbol{Q}$ be $\boldsymbol{U}\boldsymbol{S}\boldsymbol{V}^{\mathrm{T}}$, where $\boldsymbol{S}^2 = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{2MT-2K})$. Assuming $\boldsymbol{U} = (\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_{2MT-2K})$ and $\boldsymbol{V} = (\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{2MT-2K})$, we can rewrite $\boldsymbol{B}_{\mathrm{MV}}$ as

$$\boldsymbol{B}_{\mathrm{MV}} = \sum_{i=1}^{2MT-2K} \frac{1}{\sqrt{\lambda_i}}\boldsymbol{v}_i\boldsymbol{u}_i^{\mathrm{T}}\boldsymbol{R}^{\frac{1}{2}}\boldsymbol{A}_1/\|\boldsymbol{H}_1\|_F^2 \triangleq \sum_{i=1}^{2MT-2K} \boldsymbol{B}_i. \tag{16}$$

Then we define the shrinkage estimator of $\boldsymbol{B}$ as

$$\hat{\boldsymbol{B}} = \sum_{i=1}^{2MT-2K} \xi_i \boldsymbol{B}_i \tag{17}$$

where ξ_i are shrinkage factors which, for example, can be chosen by minimizing the MSE of $\hat{\boldsymbol{B}}$. Note that the proposed shrinkage estimator has $2MT - 2K$ degrees of freedom which are much more than the conventional DL approach, thus we can expect it give better performance than DL-based methods.

B. Minimized MSE Shrinkage Estimate

In this section, we consider the MSE minimization problem. The MSE of $\hat{\boldsymbol{B}}$ can be formulated by

$$\begin{aligned}
\mathrm{MSE}(\hat{\boldsymbol{B}}) &= \mathrm{tr}\left\{\mathrm{E}\left[\left(\hat{\boldsymbol{B}} - \mathrm{E}(\hat{\boldsymbol{B}})\right)\left(\hat{\boldsymbol{B}} - \mathrm{E}(\hat{\boldsymbol{B}})\right)^{\mathrm{T}}\right]\right\} \\
&\quad + \left\|\mathrm{E}(\hat{\boldsymbol{B}}) - \boldsymbol{B}\right\|_F^2 \\
&= \sigma^2 \mathrm{tr}\left\{\boldsymbol{V}\mathrm{diag}\left(\frac{\xi_1^2}{\lambda_1}, \ldots, \frac{\xi_{2MT-2K}^2}{\lambda_{2MT-2K}}\right)\boldsymbol{V}^{\mathrm{T}}\right\}
\end{aligned}$$

$$+\left\|\boldsymbol{V}^{\mathrm{T}}\mathrm{E}(\hat{\boldsymbol{B}}-\boldsymbol{B})\right\|_F^2$$
$$=\sum_{i=1}^{2MT-2K}\left\{\frac{\sigma^2}{\lambda_i}\xi_i^2+(1-\xi_i)^2\left\|\boldsymbol{V}^{\mathrm{T}}\mathrm{E}(\boldsymbol{B}_i)\right\|_F^2\right\}. \quad (18)$$

By solving the minimization problem, we can get the minimized MSE shrinkage factors as

$$\xi_i=\frac{\left\|\boldsymbol{V}^{\mathrm{T}}\mathrm{E}(\boldsymbol{B}_i)\right\|_F^2}{\left\|\boldsymbol{V}^{\mathrm{T}}\mathrm{E}(\boldsymbol{B}_i)\right\|_F^2+\sigma^2\Big/\lambda_i}=\frac{\lambda_i}{\lambda_i+\sigma^2\Big/\left\|\boldsymbol{V}^{\mathrm{T}}\mathrm{E}(\boldsymbol{B}_i)\right\|_F^2}. \quad (19)$$

In practice, since both σ^2 and $\|\boldsymbol{V}^{\mathrm{T}}\mathrm{E}(\boldsymbol{B}_i)\|_F^2$ are unknown, we will use

$$\hat{\sigma}^2=\left\|\boldsymbol{R}^{\frac{1}{2}}\frac{\boldsymbol{A}_1}{\|\boldsymbol{H}_1\|_F^2}-\boldsymbol{R}^{\frac{1}{2}}\boldsymbol{Q}\boldsymbol{B}_{\mathrm{MV}}\right\|_F^2 \quad (20)$$

as an estimate of σ^2 and $\|\boldsymbol{V}^{\mathrm{T}}\boldsymbol{B}_i\|_F^2$ instead of $\|\boldsymbol{V}^{\mathrm{T}}\mathrm{E}(\boldsymbol{B}_i)\|_F^2$.

IV. Automatic Robust Minimum Variance Receiver

Using the shrinkage factors in (19), we can get an automatic robust receiver via generalized loading. Denoting the diagonal matrix

$$\boldsymbol{\Xi}=\mathrm{diag}\left\{\frac{\sigma^2}{\left\|\boldsymbol{V}^{\mathrm{T}}\mathrm{E}(\boldsymbol{B}_1)\right\|_F^2},\ldots,\frac{\sigma^2}{\left\|\boldsymbol{V}^{\mathrm{T}}\mathrm{E}(\boldsymbol{B}_{2MT-2K})\right\|_F^2}\right\} \quad (21)$$

we can compute the coefficient matrix $\hat{\boldsymbol{W}}$ as

$$\begin{aligned}\hat{\boldsymbol{W}}&=\boldsymbol{A}_1/\|\boldsymbol{H}_1\|_F^2-\boldsymbol{Q}\hat{\boldsymbol{B}}\\&=\frac{\boldsymbol{A}_1}{\|\boldsymbol{H}_1\|_F^2}\\&\quad-\boldsymbol{Q}\sum_{i=1}^{2MT-2K}\frac{\sqrt{\lambda_i}}{\lambda_i+\frac{\sigma^2}{\|\boldsymbol{V}^{\mathrm{T}}\mathrm{E}(\boldsymbol{B}_i)\|_F^2}}\boldsymbol{v}_i\boldsymbol{u}_i^{\mathrm{T}}\boldsymbol{R}^{\frac{1}{2}}\frac{\boldsymbol{A}_1}{\|\boldsymbol{H}_1\|_F^2}\\&=\left[\boldsymbol{I}-\boldsymbol{Q}(\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q}+\boldsymbol{V}\boldsymbol{\Xi}\boldsymbol{V}^{\mathrm{T}})^{-1}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\right]\boldsymbol{A}_1/\|\boldsymbol{H}_1\|_F^2. \quad (22)\end{aligned}$$

Since $\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{A}_1=\boldsymbol{0}$, we add

$$-\boldsymbol{Q}(\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q}+\boldsymbol{V}\boldsymbol{\Xi}\boldsymbol{V}^{\mathrm{T}})^{-1}\boldsymbol{V}\boldsymbol{\Xi}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{A}_1/\|\boldsymbol{H}_1\|_F^2=\boldsymbol{0} \quad (23)$$

to (22) and define $\tilde{\boldsymbol{R}}=\boldsymbol{R}+\boldsymbol{Q}\boldsymbol{V}\boldsymbol{\Xi}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}}$. Then we get

$$\begin{aligned}&\left[\boldsymbol{I}-\boldsymbol{Q}(\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q}+\boldsymbol{V}\boldsymbol{\Xi}\boldsymbol{V}^{\mathrm{T}})^{-1}\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{R}+\boldsymbol{Q}\boldsymbol{V}\boldsymbol{\Xi}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}})\right]\frac{\boldsymbol{A}_1}{\|\boldsymbol{H}_1\|_F^2}\\&=\tilde{\boldsymbol{R}}^{-\frac{1}{2}}\left[\boldsymbol{I}-\tilde{\boldsymbol{R}}^{\frac{1}{2}}\boldsymbol{Q}(\boldsymbol{Q}^{\mathrm{T}}\tilde{\boldsymbol{R}}\boldsymbol{Q})^{-1}\boldsymbol{Q}^{\mathrm{T}}\tilde{\boldsymbol{R}}^{\frac{1}{2}}\right]\tilde{\boldsymbol{R}}^{\frac{1}{2}}\boldsymbol{A}_1/\|\boldsymbol{H}_1\|_F^2. \quad (24)\end{aligned}$$

We can see that $\boldsymbol{I}-\tilde{\boldsymbol{R}}^{1/2}\boldsymbol{Q}(\boldsymbol{Q}^{\mathrm{T}}\tilde{\boldsymbol{R}}\boldsymbol{Q})^{-1}\boldsymbol{Q}^{\mathrm{T}}\tilde{\boldsymbol{R}}^{1/2}$ is the orthogonal projection matrix onto the complement of the column space of $\tilde{\boldsymbol{R}}^{1/2}\boldsymbol{Q}$. Note that $\tilde{\boldsymbol{R}}^{1/2}\boldsymbol{Q}$ has rank $2MT-2K$ and $\tilde{\boldsymbol{R}}^{-1/2}\boldsymbol{A}_1$ with rank $2K$ satisfies

$$\left(\tilde{\boldsymbol{R}}^{\frac{1}{2}}\boldsymbol{Q}\right)^{\mathrm{T}}\tilde{\boldsymbol{R}}^{-\frac{1}{2}}\boldsymbol{A}_1=\boldsymbol{0}. \quad (25)$$

Consequently

$$\begin{aligned}\hat{\boldsymbol{W}}&=\tilde{\boldsymbol{R}}^{-\frac{1}{2}}\left[\tilde{\boldsymbol{R}}^{-\frac{1}{2}}\boldsymbol{A}_1(\boldsymbol{A}_1^{\mathrm{T}}\tilde{\boldsymbol{R}}^{-1}\boldsymbol{A}_1)^{-1}\boldsymbol{A}_1^{\mathrm{T}}\tilde{\boldsymbol{R}}^{-\frac{1}{2}}\right]\tilde{\boldsymbol{R}}^{\frac{1}{2}}\boldsymbol{A}_1/\|\boldsymbol{H}_1\|_F^2\\&=(\boldsymbol{R}+\boldsymbol{Q}\boldsymbol{V}\boldsymbol{\Xi}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}})^{-1}\boldsymbol{A}_1\\&\quad\times\left[\boldsymbol{A}_1^{\mathrm{T}}(\boldsymbol{R}+\boldsymbol{Q}\boldsymbol{V}\boldsymbol{\Xi}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}})^{-1}\boldsymbol{A}_1\right]^{-1}. \quad (26)\end{aligned}$$

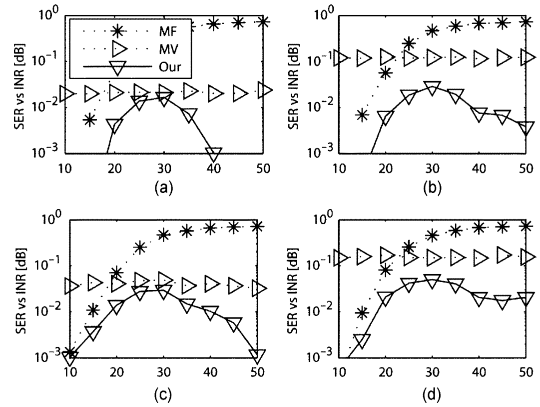

Fig. 1. SER for varying INR ($\mathrm{SNR}=20$ dB). (a) $\mathrm{J}=80$, perfect CSI; (b) $\mathrm{J}=40$, perfect CSI; (c) $\mathrm{J}=80$, $\sigma_e^2=0.1$; (d) $\mathrm{J}=40$, $\sigma_e^2=0.1$.

From (26), we can see that the proposed automatic robust receiver can be seen as a generalized loading approach in which symmetric matrix $\boldsymbol{Q}\boldsymbol{V}\boldsymbol{\Xi}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}}$ is loaded instead of diagonal matrix. Furthermore, we also note that the conventional diagonal loading approach can be obtained as a special case of the shrinkage estimator of (17) by choosing $\xi_i=\lambda_i/(\lambda_i+\rho)$, where ρ is the parameter of diagonal load.

The computational complexity of the proposed method is mainly determined by the eigenvalue decomposition of $\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{Q}$, which is only $\mathcal{O}[(2MT-2K)^3]$. Thus, the proposed method has about the same computational complexity as the MV receiver. However, other existing robust methods usually have much higher complexity and may also requires a specific built-in convex optimization software, such as [8], [9].

V. Numerical Examples

Throughout the simulations, we assume a single receiver of $M=4$ antennas. The interfering transmitter uses the same OSTBC as the transmitter of interest, and the QPSK modulation scheme is used. The methods we evaluate include a) the MF receiver, b) the MV receiver, c) the DL-based MV receiver with a loading factor equals to five times of the noise variance which is a popular *ad hoc* choice, d) the proposed robust MV receiver, and e) the worst-case-based method [8] (one of the robust methods with demands of user parameters). We assume the knowledge of mismatches (the variance) on channel matrices is known for it and can be used for choosing the user parameter. All plots are averaged over 100 independent simulation runs. In each run, the elements of $\boldsymbol{H}_p$ are independently drawn from a complex Gaussian random generator with zero mean and unit variance and the presumed channel matrices are perturbed by a complex Gaussian noise which is independently generated for each channel matrix with zero mean and the variance σ_e^2.

In the first two examples, $P=2$ transmitters each with $N=2$ antennas are assumed, and the full-rate Alamouti's OSTBC ($T=2, K=2$) [1] is used. First, we assume the $\mathrm{SNR}=20$ dB. The symbol error rate (SER) performance for varying INR in different cases is shown in Fig. 1(a)–(d). Note that INR stands for interference-to-noise ratio. When INR increases the interference from other users dominates the noise. The MF approach does not take into account any interference. Therefore the performance deteriorates as INR increases. According to the plots, we can see that the MF receiver fails completely when INR is high. The MV receiver can perform well in

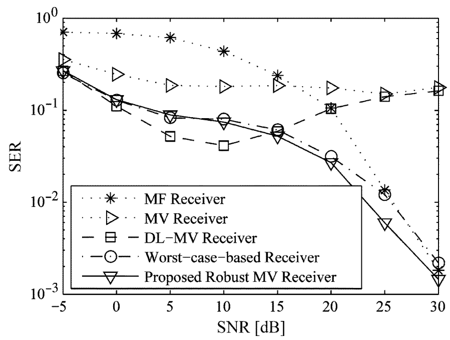

Fig. 2. SER for varying SNR ($J = 40, \sigma_e^2 = 0.2, \mathrm{INR} = 20$ dB).

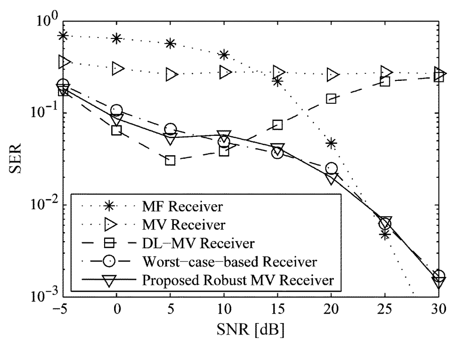

Fig. 3. SER for varying SNR ($J = 60, \sigma_e^2 = 0.2, \mathrm{INR} = 20$ dB).

the existing of MAI when perfect CSI is known and sample sizes are high. However, in case of imperfect CSI and/or low sample sizes, the performance of MV receiver degrades severely. In all the cases, the proposed robust MV receiver gives robust performance as compared with the MV receiver and the MF receiver. Next, we assume the INR is fixed to 20 dB. The SERs of all the tested receivers for varying SNR when $J = 40, \sigma_e^2 = 0.2$ (about -7 dB) are shown in Fig. 2. The user parameter $2\sigma_e$ (which is nearly optimal) is used for the worst-case-based method. We can see that the proposed method gives good performance in a wide range of SNR, which is similar to that of the worst-case-based method. In the next two examples, we assume $P = 2$ transmitters each with $N = 3$ antennas, and the 3/4-rate $(K = 3, T = 4)$ orthogonal design STBC from [2] is used. The INR is fixed to 20 dB, and the user parameter $2\sigma_e$ (which is nearly optimal for this example) is taken for the method based on worst-case optimization. The receiver SERs versus the SNR for $J = 60$ are shown in Fig. 3. Fig. 4 shows the SERs versus the number of sample data blocks when $\mathrm{SNR} = 20$ dB. According to the results, we can see that the MV and DL-MV receivers are very sensitive to sample sizes. However, the proposed method is robust to errors on sample covariance matrix. It shows similar SER performance to the worst-case-based method.

VI. Conclusion

In this letter, we derive a robust receiver based on the MV linear receiver. The proposed method is parameter-free and has a closed form which can be computed efficiently. Simulations

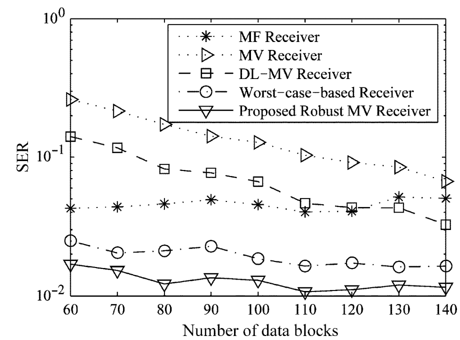

Fig. 4. SER for varying J ($\sigma_e^2 = 0.2, \mathrm{SNR} = \mathrm{INR} = 20$ dB).

show that our receiver gives robust performance against CSI mismatches even in case of low sample sizes. Note that there are still some opportunities to enhance the proposed method, for example, by exploiting the structure of OSTBC (see [12]) and/or by taking into account the random effect of the designed matrix in the linear regression model of (14). We plan to discuss these issues in further work.

References

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, 1998.

[2] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1456–1467, 1999.

[3] G. Ganesan and P. Stoica, "Space-time block codes: A maximum SNR approach," *IEEE Trans. Inf. Theory*, vol. 47, no. 4, pp. 1650–1656, 2001.

[4] A. F. Naguib, N. Seshadri, and A. Calderbank, "Increasing data rate over wireless channels," *IEEE Signal Process. Mag.*, vol. 17, no. 3, pp. 76–92, 2000.

[5] H. Li, X. Lu, and G. B. Giannakis, "Capon multiuser receiver for CDMA systems with space-time coding," *IEEE Trans. Signal Process.*, vol. 50, no. 5, pp. 1193–1204, 2002.

[6] D. Reynolds, X. Wang, and H. V. Poor, "Blind adaptive space-time multiuser detection with multiple transmitter and receiver antennas," *IEEE Trans. Signal Process.*, vol. 50, no. 6, pp. 1261–1276, 2002.

[7] S. Shahbazpanahi, M. Beheshti, A. B. Gershman, M. Gharavi-Alkhansari, and K. M. Wong, "Minimum variance linear receivers for multiaccess MIMO wireless systems with space-time block coding," *IEEE Trans. Signal Process.*, vol. 52, no. 12, pp. 3306–3313, 2004.

[8] Y. Rong, S. Shahbazpanahi, and A. B. Gershman, "Robust linear receivers for space-time block coded multiaccess MIMO systems with imperfect channel state information," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3081–3090, 2005.

[9] Y. Rong, S. A. Vorobyov, and A. B. Gershman, "Robust linear receivers for multiaccess space-time block-coded MIMO systems: A probabilistically constrained approach," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1560–1569, 2006.

[10] R. L. G. Cavalcante and I. Yamada, "Multiaccess interference suppression in orthogonal space-time block coded MIMO systems by adaptive projected subgradient method," *IEEE Trans. Signal Process.*, vol. 56, no. 3, pp. 1028–1042, 2008.

[11] J. Yang, X. Ma, C. Hou, and Y. Liu, "Automatic generalized loading for robust adaptive beamforming," *IEEE Signal Process. Lett.*, vol. 16, no. 3, pp. 219–222, Mar. 2009.

[12] Y. Rong, S. Shahbazpanahi, and A. B. Gershman, "Exploiting the structure of OSTBC's to improve the robustness of worst-case optimization based linear multi-user MIMO receivers," in *Proc. ICASSP'05*, Mar. 2005, pp. 781–784.