Binomial and Poisson Distributions

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Binomial Distribution

- $Binomial(n, \theta)$:
  - Out of n independent trials, each of which has a probability θ of success, what is the probability of a total of k successes?
  - Example: If a coin is tossed n times, and has a probability θ of coming up heads, what is the probability that it will come up heads
- If $K = B_1 + B_2 + \cdots + B_n, B_i \sim Bernoulli(\theta)$, iid, then $K \sim Binomial(n, \theta)$

B_i is 1 if trial succeeds, 0 if it fails. The sum of all the B's is the number of successes

Binomial Distribution

- $K \sim Binomial(n, \theta)$ then
$$\mathrm{p}(K = k) = \binom{n}{k} \theta^k (1 - \theta)^{1-k}, \binom{n}{k} = \frac{n!}{k!(n-k)!}$$
- (Generate Binomial variables in R)

Poisson Distribution

- Appropriate for modelling the number of events that occur within a certain time interval
  - E.g.: number of goals in a soccer match
- Appropriate when:
  - The events are independent of each other
  - The expected number of events is proportional to the length of the time interval

Poisson Distribution

- $K \sim Poisson(\lambda) \Rightarrow P_\lambda(K = k) = \frac{\lambda^k \exp(-\lambda)}{k!}$
- $E(K) = \lambda$, the average number of events in an interval
- $Var(K) = \lambda$
- For large λ, $Poisson(\lambda)$ is approximately $N(\lambda, \lambda)$

Poisson vs. Binomial Distribution

- $Binomial(n, \theta)$: the total number of successes is bounded by n
- $Poisson(\lambda)$: the events are independent, the number of events is *not* bounded