Two-Way ANOVA
Case Study: The Pygmalion Effect

Jean-Baptiste Regnault, *Pygmalion* (1786)

Some slides from Craig Burkett

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The Pygmalion Effect in Psychology

- In Greek mythology, Pygmalion was a sculptor who fell in love with a statue he made
- In Psychology: if a supervisor has high expectations of the people they supervise, the performance improves
  - E.g., students perform better in classes where the professor was told the students are good
  - Is the performance different or just the evaluation?

Pygmalion Effect Experiment

- Army training camp with 10 companies (each company has about 100 soldiers)
  - Each company is divided up into 3 platoons
- One platoon is randomly chosen to be the “Pygmalion” group in each company
  - The two others are control platoons
- Prior to training, the platoon sergeant was told that his platoon was superior
  - But actually, platoons were assigned randomly to control/Pygmalion effect
- At the end, the performance of each platoon was evaluated (not by its own sergeant) on how they operate weapons

Visualizing the data

- (in R)

Two-way ANOVA Terminology

- One (numerical) response variable
  - Dependent, Outcome
- Two categorical independent variables
  - Treatments, Predictors, Explanatory
- If the independent variables are crossed (i.e., we have observations for all/most combinations of levels of different variables), the experiment is said to have a **Factorial Design**
  - If there are observations at each treatment combination, called a **complete** design

Experimental Units

- A minimal unit that could possibly receive a unique treatment
  - In this situation, a platoon
  - Can imagine measuring each soldier individually, with groups of soldiers being either Pygmalion or Control, but each soldier also receiving his own treatment (possibly unknown)
    - Would have to be careful there – as it stands, the observations for each soldier are *not* independent, so we couldn’t just run a regression model without having platoons as variables
    - Will hopefully talk about this later in the course

Multiple Linear Regression Model

- 10 companies, 2 treatments
- Set up indicator variables:
  - 1 for treatment
  - 9 for companies
  - 9 for interaction terms
- Note: we have *barely* enough data to estimate the full model
  - What if we had just one measurement per treatment x company?
- Model:
$Y_i = \beta_0 + \beta_1 I_{Pyg,i} + \beta_2 I_{Comp2,i} + \cdots + \beta_{10} I_{Comp10,i} + \beta_{11} I_{Pyg,i} \cdot I_{Comp2,i} + \cdots + \beta_{19} I_{Pyg,i} \cdot I_{Comp10,i} + \varepsilon_i$

Multiple Linear Regression Model

- $Y_i = \beta_0 + \beta_1 I_{Pyg,i} + \beta_2 I_{Comp2,i} + \cdots + \beta_{10} I_{Comp10,i} + \beta_{11} I_{Pyg,i} \cdot I_{Comp2,i} + \cdots + \beta_{19} I_{Pyg,i} \cdot I_{Comp10,i} + \varepsilon_i$

- What’s the mean for Company 1, with the Pygmalion treatment?

- What’s the mean for Company 2, without the Pygmalion treatment?

Interactions

- $Y_i = \beta_0 + \beta_1 I_{Pyg,i} + \beta_2 I_{Comp2,i} + \cdots + \beta_{10} I_{Comp10,i} + \beta_{11} I_{Pyg,i} \cdot I_{Comp2,i} + \cdots + \beta_{19} I_{Pyg,i} \cdot I_{Comp10,i} + \varepsilon_i$

- What do interaction effects mean here?
  - Non-zero interaction terms: in some companies, the Pygmalion effect helps a lot. In some companies, it doesn’t help at all

Fitting the Full Model

- (in R)

Partial F-test for the Full Model

```
> anova(fit_saturated)
Analysis of Variance Table

Response: Score
               Df Sum Sq Mean Sq F value  Pr(>F)
Treat           1 327.34  327.34  6.3080 0.03323 *
Company         9 682.52   75.84  1.4614 0.29051
Treat:Company   9 311.46   34.61  0.6669 0.72212
Residuals       9 467.04   51.89
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Can compute the *unexplained variance* due to each component. E.g.

$$\sum_i (Score_i - \widehat{Score}_i^{Full})^2 - \sum_i (Score_i - \widehat{Score}_i^{NoTreatment})^2 = 327.34,$$

$$F = \frac{\left(\frac{327.34}{1}\right)}{\left(\frac{467.04}{9}\right)}$$

$F \sim F(1, 9)$ if $\beta_{pyg} = 0$

Partial F-test for the Full Model

- Small p-value -> Can reject the hypothesis that the coefficient is 0
- For this kind of experiment, it’s arguably okay to discard the non-significant factors
  - They might still matter
  - Decreases the p-value (why?)

Interactions

- We have 2 treatments, and 10 companies
  - For the Pygmalion platoons, we have a simple effect of company on shooting score
  - For the Control platoons, we have another simple effect of company on shooting score
- These two simple effects, averaged together, are called the **main effect** of company
  - If the simple effects are the same as the main effect, then there is no **interaction** present

Hypothetical Experiments

- 8 experiments, each involving 2 levels of 2 different factors (A and B)

Exp't 1			
	b_1	b_2	$\bar{Y}_A$
a_1	**5**	**5**	5
a_2	**5**	**5**	5
$\bar{Y}_B$	5	5	

Exp't 2			
	b_1	b_2	$\bar{Y}_A$
a_1	**4**	**4**	4
a_2	**6**	**6**	6
$\bar{Y}_B$	5	5	

Exp't 5			
	b_1	b_2	$\bar{Y}_A$
a_1	**6**	**4**	5
a_2	**4**	**6**	5
$\bar{Y}_B$	5	5	

Exp't 6			
	b_1	b_2	$\bar{Y}_A$
a_1	**5**	**3**	4
a_2	**5**	**7**	6
$\bar{Y}_B$	5	5	

Exp't 3			
	b_1	b_2	$\bar{Y}_A$
a_1	**7**	**3**	5
a_2	**7**	**3**	5
$\bar{Y}_B$	7	3	

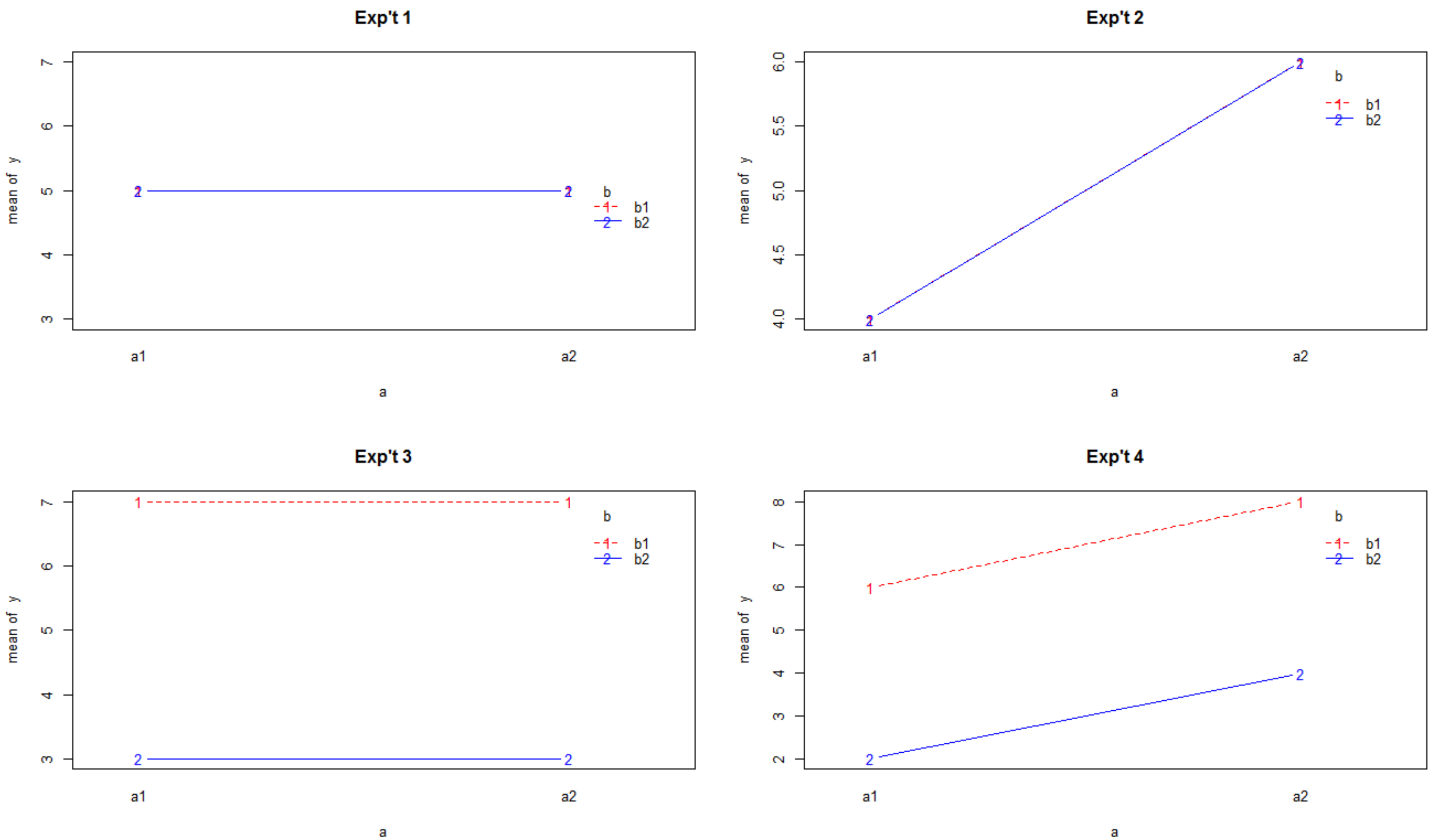
Exp't 4			
	b_1	b_2	$\bar{Y}_A$
a_1	**6**	**2**	4
a_2	**8**	**4**	6
$\bar{Y}_B$	7	3	

Exp't 7			
	b_1	b_2	$\bar{Y}_A$
a_1	**8**	**2**	5
a_2	**6**	**4**	5
$\bar{Y}_B$	7	3	

Exp't 8			
	b_1	b_2	$\bar{Y}_A$
a_1	**7**	**1**	4
a_2	**7**	**5**	6
$\bar{Y}_B$	7	3	

No interaction

- Parallel lines



Interactions present