Linear Regression & Pooled t-test

$$b_1 = \frac{\sum(x_i-\overline{x})(y_i-\overline{y})}{\sum(x_i-\overline{x})^2} = \frac{\sum x_i y_i - n\overline{x}\overline{y}}{\sum x_i^2 - n\overline{x}^2} \qquad b_0 = \overline{y} - b_1\overline{x} \qquad t_{obs} = \frac{\overline{y}_1 - \overline{y}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

One-way analysis of variance

$$\text{SST} = \sum_{i,j=1}^{N}(y_{ij}-\overline{y})^2 \qquad \text{SSE} = \sum_{i,j=1}^{N}(y_{ij}-\overline{y}_i)^2 \qquad \text{SSR} = \sum_{i,j=1}^{N}(\overline{y}_i-\overline{y})^2$$

Bernoulli and Binomial distributions

If $Y \sim \text{Bernoulli}(\pi)$
$\text{E}(Y) = \pi$, $\text{Var}(Y) = \pi(1-\pi)$

If $Y \sim \text{Binomial}(m, \pi)$
$\text{E}(Y) = m\pi$, $\text{Var}(Y) = m\pi(1-\pi)$

Poisson Distribution

If $Y \sim Poisson(\lambda)$
$E(Y) = \lambda$

$P(Y = k) = \frac{\lambda^k \exp(-\lambda)}{k!}$
$Var(Y) = \lambda$

Logistic Regression with Binomial Response formulae

$$\text{Deviance} = const - 2\log P(y|\beta)$$

$$\text{AIC} = -2\log(L) + 2(p) \qquad \text{LRT} = 2\log(LMAX_{full}) - 2\log(LMAX_{reduced})$$

Common Distributions

$$Z_1, Z_2, ..., Z_k \sim N(0,1) \text{ i.i.d, } W = Z_1^2 + ... + Z_k^2 \Rightarrow W \sim \chi^2(k)$$

$$Z \sim N(0,1), W \sim \chi^2(k) \text{ indep., } T = \frac{Z}{\sqrt{W/k}} \sim t(k)$$

The function `pnorm(q, mean, sd)` computes the proportion of observations in the normal distribution that are less than or equal to q; that is, it computes $P(X \leq q)$, where $X \sim N(\mu, \sigma)$.

The function `qnorm(p, mean, sigma)` computes the quantile for which there is a probability of p of getting a value less than or equal to it. Thus, the quantile is the value q such that $P(X \leq q) = p$ for a given p. In other words, `qnorm` converts proportions to quantiles while pnorm converts quantiles to proportions which means that qnorm and pnorm are inverse functions of each other.

The functions `pchisq(x, df)` and works similarly to `pnorm`, except for the χ^2 distribution with *df* degrees of freedom

The functions `qt(q, df)` and `pt(q, df)` and the functions `pf(q, df1, df2)` and `qf(p, df1, df2)` work similarly to `qnorm` and `pnorm`, but for the F and Q distributions.

```
set.seed(0)
p <- c()

sig_t_test <- c()

n_sims <- 1000
mu <- 60
sigma <- 15
n_pairs <- 14

for (i in 1:n_sims){
  dat <- data.frame(matrix(ncol=2, nrow=0))
  colnames(dat) <- c("Pair", "Percentage")

  new_dat <- data.frame(matrix(ncol=2, nrow=1))
  colnames(new_dat) <- c("Pair", "Percentage")


  pair <- "Pair1"
  for (i in 1:n_pairs){
    percentage <- rnorm(1, mean=mu, sd=sigma)
    new_dat["Pair"] <- pair
    new_dat["Percentage"] <- percentage
    dat <- rbind(dat, new_dat)
  }

  pair <- "Pair2"
  for (i in 1:n_pairs){
    percentage <- rnorm(1, mean=mu, sd=sigma)
    new_dat["Pair"] <- pair
    new_dat["Percentage"] <- percentage
    dat <- rbind(dat, new_dat)
  }
  ttestpvals <-with(dat, pairwise.t.test(Percentage, Pair, p.adj = "none"))$p.value
  sig_t_test <- c(sig_t_test, sum(ttestpvals<0.05, na.rm=TRUE)>0)
}
```

```
sim <- function(n_pairs, n_obs, mu, sigma){
  dat <- data.frame(Pair = paste('Pair', rep(1:n_pairs, each = n_obs), sep = ''),
                    Percentage = rnorm(n_pairs*n_obs, mu, sigma))
  ttestpvals <-with(dat, pairwise.t.test(Percentage, Pair, p.adj = "none"))$p.value
  sig_t_tests <- any(ttestpvals < 0.05, na.rm = TRUE)
  return(sig_t_tests)
}

n_sims <- 1000
n_pairs <- 6
n_obs <- 14
mu <- 60
sigma <- 15
t_test_rej <- replicate(n_sims, sim(n_pairs, n_obs, mu, sigma))
mean(t_test_rej)*100
```

Critical values of the χ^2 distribution. Upper tail area is across the top.

DF	0.995	0.99	0.975	0.95	0.9	0.5	0.1	0.05	0.025	0.01	0.005
1	0.0	0.0	0.0	0.0	0.0	0.5	2.7	3.8	5.0	6.6	7.9
2	0.0	0.0	0.1	0.1	0.2	1.4	4.6	6.0	7.4	9.2	10.6
3	0.1	0.1	0.2	0.4	0.6	2.4	6.3	7.8	9.3	11.3	12.8
4	0.2	0.3	0.5	0.7	1.1	3.4	7.8	9.5	11.1	13.3	14.9
5	0.4	0.6	0.8	1.1	1.6	4.4	9.2	11.1	12.8	15.1	16.7
6	0.7	0.9	1.2	1.6	2.2	5.3	10.6	12.6	14.4	16.8	18.5
7	1.0	1.2	1.7	2.2	2.8	6.3	12.0	14.1	16.0	18.5	20.3
8	1.3	1.6	2.2	2.7	3.5	7.3	13.4	15.5	17.5	20.1	22.0
9	1.7	2.1	2.7	3.3	4.2	8.3	14.7	16.9	19.0	21.7	23.6
10	2.2	2.6	3.2	3.9	4.9	9.3	16.0	18.3	20.5	23.2	25.2
11	2.6	3.1	3.8	4.6	5.6	10.3	17.3	19.7	21.9	24.7	26.8
12	3.1	3.6	4.4	5.2	6.3	11.3	18.5	21.0	23.3	26.2	28.3
13	3.6	4.1	5.0	5.9	7.0	12.3	19.8	22.4	24.7	27.7	29.8
14	4.1	4.7	5.6	6.6	7.8	13.3	21.1	23.7	26.1	29.1	31.3
16	5.1	5.8	6.9	8.0	9.3	15.3	23.5	26.3	28.8	32.0	34.3
18	6.3	7.0	8.2	9.4	10.9	17.3	26.0	28.9	31.5	34.8	37.2
20	7.4	8.3	9.6	10.9	12.4	19.3	28.4	31.4	34.2	37.6	40.0
24	9.9	10.9	12.4	13.8	15.7	23.3	33.2	36.4	39.4	43.0	45.6
28	12.5	13.6	15.3	16.9	18.9	27.3	37.9	41.3	44.5	48.3	51.0
32	15.1	16.4	18.3	20.1	22.3	31.3	42.6	46.2	49.5	53.5	56.3
36	17.9	19.2	21.3	23.3	25.6	35.3	47.2	51.0	54.4	58.6	61.6
40	20.7	22.2	24.4	26.5	29.1	39.3	51.8	55.8	59.3	63.7	66.8
50	28.0	29.7	32.4	34.8	37.7	49.3	63.2	67.5	71.4	76.2	79.5
60	35.5	37.5	40.5	43.2	46.5	59.3	74.4	79.1	83.3	88.4	92.0
70	43.3	45.4	48.8	51.7	55.3	69.3	85.5	90.5	95.0	100.4	104.2
80	51.2	53.5	57.2	60.4	64.3	79.3	96.6	101.9	106.6	112.3	116.3
100	67.3	70.1	74.2	77.9	82.4	99.3	118.5	124.3	129.6	135.8	140.2
150	109.1	112.7	118.0	122.7	128.3	149.3	172.6	179.6	185.8	193.2	198.4
200	152.2	156.4	162.7	168.3	174.8	199.3	226.0	234.0	241.1	249.4	255.3
300	240.7	246.0	253.9	260.9	269.1	299.3	331.8	341.4	349.9	359.9	366.8
400	330.9	337.2	346.5	354.6	364.2	399.3	436.6	447.6	457.3	468.7	476.6
500	422.3	429.4	439.9	449.1	459.9	499.3	540.9	553.1	563.9	576.5	585.2
600	514.5	522.4	534.0	544.2	556.1	599.3	644.8	658.1	669.8	683.5	693.0
700	607.4	615.9	628.6	639.6	652.5	699.3	748.4	762.7	775.2	790.0	800.1
800	700.7	709.9	723.5	735.4	749.2	799.3	851.7	866.9	880.3	896.0	906.8
900	794.5	804.3	818.8	831.4	846.1	899.3	954.8	970.9	985.0	1001.6	1013.0
1000	888.6	898.9	914.3	927.6	943.1	999.3	1057.7	1074.7	1089.5	1107.0	1118.9

NORMAL CUMULATIVE DISTRIBUTION FUNCTION

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000