STA2101 Formulas

Columns of $\mathbf{A}$ *linearly dependent* means there is a vector $\mathbf{v} \neq \mathbf{0}$ with $\mathbf{A}\mathbf{v} = \mathbf{0}$.

Columns of $\mathbf{A}$ *linearly independent* means that $\mathbf{A}\mathbf{v} = \mathbf{0}$ implies $\mathbf{v} = \mathbf{0}$.

$\mathbf{A}$ *positive definite* means $\mathbf{v}^\top \mathbf{A}\mathbf{v} > 0$ for all vectors $\mathbf{v} \neq \mathbf{0}$.

$\boldsymbol{\Sigma} = \mathbf{P}\boldsymbol{\Lambda}\mathbf{P}^\top$

$\boldsymbol{\Sigma}^{-1} = \mathbf{P}\boldsymbol{\Lambda}^{-1}\mathbf{P}^\top$

$\boldsymbol{\Sigma}^{1/2} = \mathbf{P}\boldsymbol{\Lambda}^{1/2}\mathbf{P}^\top$

$\boldsymbol{\Sigma}^{-1/2} = \mathbf{P}\boldsymbol{\Lambda}^{-1/2}\mathbf{P}^\top$

If $\lim_{n\to\infty} E(T_n) = \theta$ and $\lim_{n\to\infty} Var(T_n) = 0$, then $T_n \stackrel{p}{\rightarrow} \theta$

If $\sqrt{n}(T_n - \mu) \stackrel{d}{\rightarrow} T \sim N(0,\sigma^2)$, then $\sqrt{n}\,(g(T_n) - g(\mu)) \stackrel{d}{\rightarrow} g'(\mu)T \sim N(0, g'(\mu)^2\sigma^2)$

If $\mathbf{T}_n \stackrel{d}{\rightarrow} \mathbf{T}$ and $\mathbf{Y}_n \stackrel{p}{\rightarrow} \mathbf{c}$, then $\left(\begin{array}{c} \mathbf{T}_n \\ \\ \mathbf{Y}_n \end{array}\right) \stackrel{d}{\rightarrow} \left(\begin{array}{c} \mathbf{T} \\ \\ \mathbf{c} \end{array}\right)$

$\sqrt{n}(\overline{\mathbf{x}}_n - \boldsymbol{\mu}) \stackrel{d}{\rightarrow} \mathbf{x} \sim N(\mathbf{0}, \boldsymbol{\Sigma})$

Let $g : \mathbb{R}^d \rightarrow \mathbb{R}^k$ etc. If $\sqrt{n}(\mathbf{T}_n - \boldsymbol{\theta}) \stackrel{d}{\rightarrow} \mathbf{T}$, then $\sqrt{n}(g(\mathbf{T}_n) - g(\boldsymbol{\theta})) \stackrel{d}{\rightarrow} \dot{\mathrm{g}}(\boldsymbol{\theta})\mathbf{T}$, where $\dot{\mathrm{g}}(\boldsymbol{\theta}) = \left[\frac{\partial g_i}{\partial \theta_j}\right]_{k\times d}$

$cov(\mathbf{w}) = E\left\{(\mathbf{w} - \boldsymbol{\mu}_w)(\mathbf{w} - \boldsymbol{\mu}_w)^\top\right\}$

$cov(\mathbf{w}, \mathbf{t}) = E\left\{(\mathbf{w} - \boldsymbol{\mu}_w)(\mathbf{t} - \boldsymbol{\mu}_t)^\top\right\}$

$cov(\mathbf{w}) = E\{\mathbf{w}\mathbf{w}^\top\} - \boldsymbol{\mu}_w\boldsymbol{\mu}_w^\top$

$cov(\mathbf{A}\mathbf{w}) = \mathbf{A}cov(\mathbf{w})\mathbf{A}^\top$

If $\mathbf{w} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then $\mathbf{A}\mathbf{w} + \mathbf{c} \sim N_r(\mathbf{A}\boldsymbol{\mu} + \mathbf{c}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)$

and $(\mathbf{w} - \boldsymbol{\mu})^\top\boldsymbol{\Sigma}^{-1}(\mathbf{w} - \boldsymbol{\mu}) \sim \chi^2(p)$

$L(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-n/2}(2\pi)^{-np/2} \exp -\frac{n}{2}\left\{tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) + (\overline{\mathbf{y}} - \boldsymbol{\mu})^\top\boldsymbol{\Sigma}^{-1}(\overline{\mathbf{y}} - \boldsymbol{\mu})\right\}$, where $\widehat{\boldsymbol{\Sigma}} = \frac{1}{n}\sum_{i=1}^n(\mathbf{y}_i - \overline{\mathbf{y}})(\mathbf{y}_i - \overline{\mathbf{y}})^\top$

$y_i = \beta_0 + \beta_1 x_{i,1} + \cdots + \beta_{p-1}x_{i,p-1} + \epsilon_i$

$\epsilon_1, \ldots, \epsilon_n$ independent $N(0, \sigma^2)$

$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$

$\boldsymbol{\epsilon} \sim N_n(\mathbf{0}, \sigma^2\mathbf{I}_n)$

$\widehat{\boldsymbol{\beta}} = (\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top\mathbf{y} \sim N_p\left(\boldsymbol{\beta}, \sigma^2(\mathbf{X}^\top\mathbf{X})^{-1}\right)$

$\widehat{\mathbf{y}} = \mathbf{X}\widehat{\boldsymbol{\beta}} = \mathbf{H}\mathbf{y}$, $\mathbf{e} = \mathbf{y} - \widehat{\mathbf{y}}$

$G^2 = -2\log\left(\frac{\max_{\theta\in\Theta_0} L(\theta)}{\max_{\theta\in\Theta} L(\theta)}\right) = -2\log\left(\frac{L(\widehat{\theta}_0)}{L(\widehat{\theta})}\right)$

$W_n = (\mathbf{L}\widehat{\boldsymbol{\theta}}_n - \mathbf{h})^\top\left(\mathbf{L}\widehat{\mathbf{V}}_n\mathbf{L}^\top\right)^{-1}(\mathbf{L}\widehat{\boldsymbol{\theta}}_n - \mathbf{h})$

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