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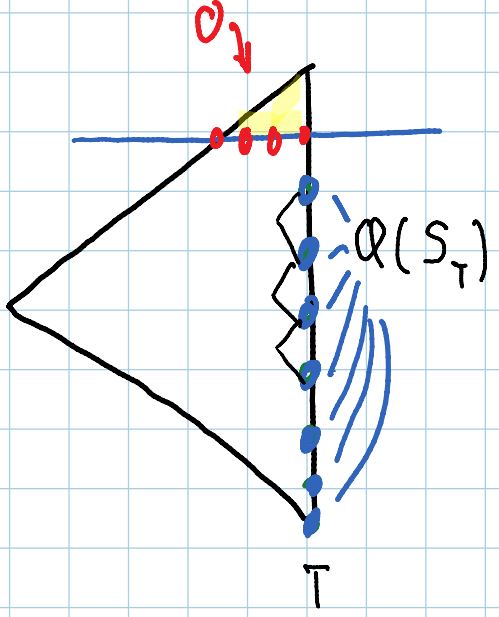
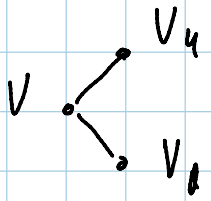
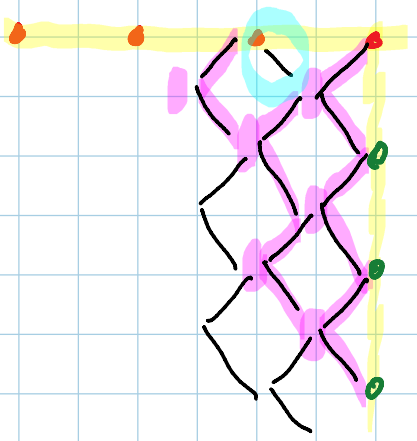
Knock-Out: get 0 upon hitting a barrier.
otherwise a std option.



$$V_{n-1} = e^{-r\Delta T}\,\mathbb{E}^{Q}\left[V_n\right]$$

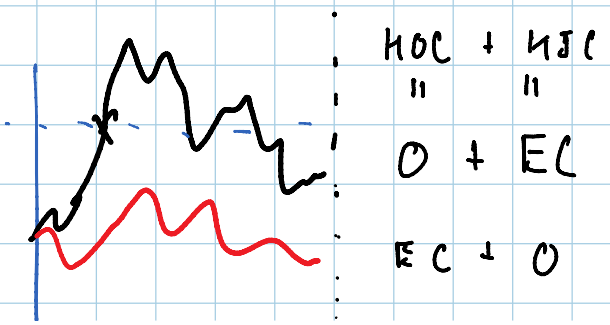




KOC = 10

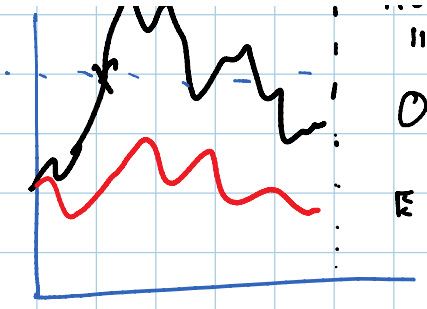
EC = 16

KIC = 6



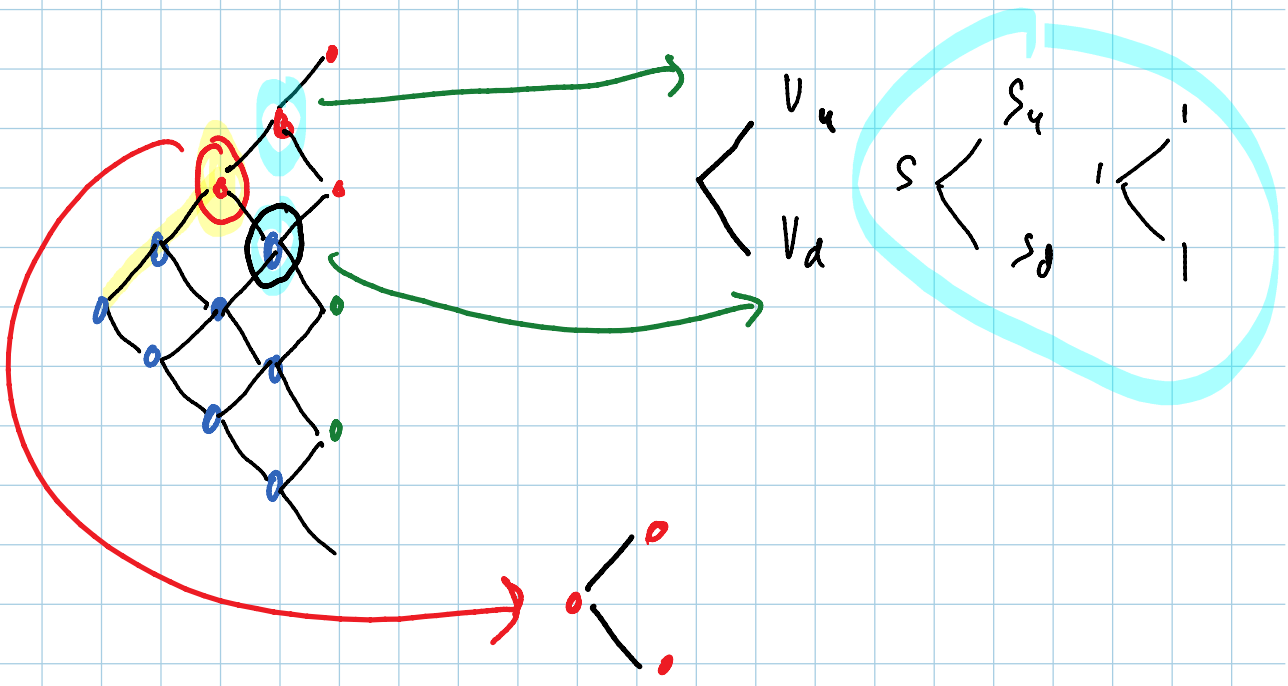
EC = 16

HzC = 6

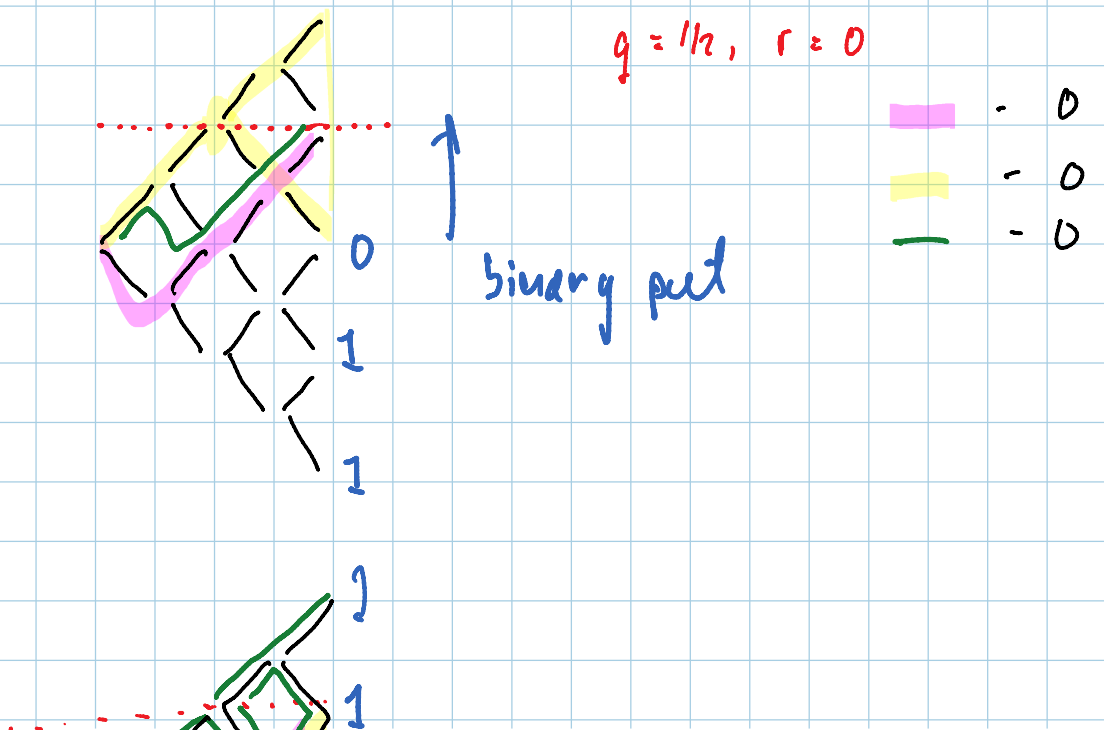


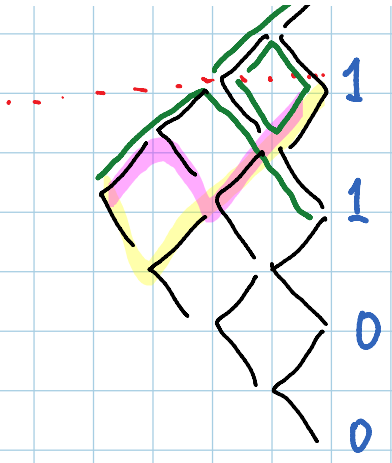
= 0 + EC

EC + 0



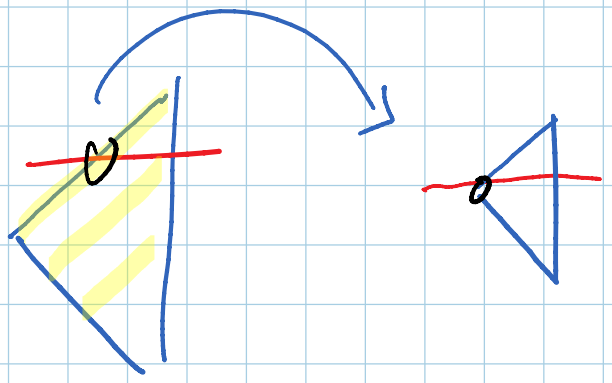
Knock-In





Binary cell

$6\left(\frac{1}{2}\right)^4$



$S_n = S_{n-1}\, e^{\sigma\sqrt{\Delta t}\, x_n}$

$u = e^{\sigma\sqrt{\Delta t}}$

$\sigma = 20\%, \quad \Delta t = \frac{1}{12}$

$r = 5\%, \quad S_0 = 100$

Interest Rates

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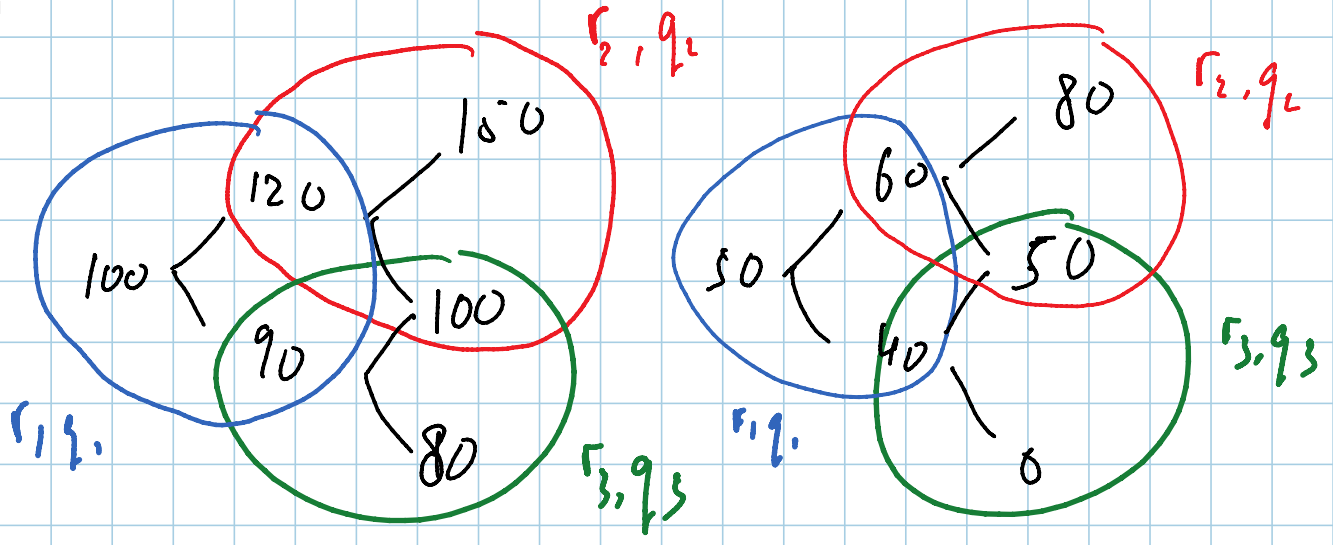
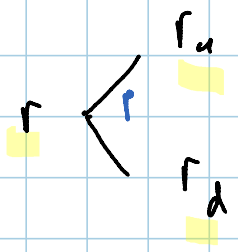
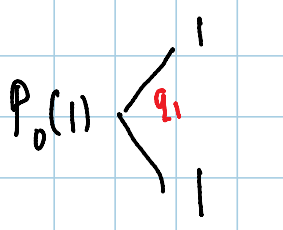
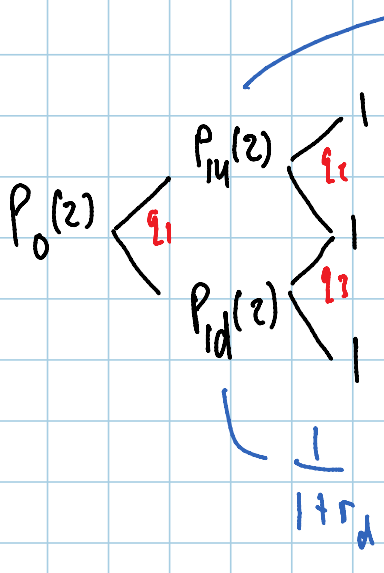




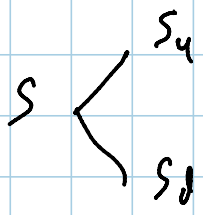
IR tree is not price of a traded asset!

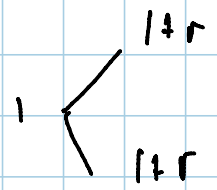




$$P_0(1) = \frac{1}{1+r}$$

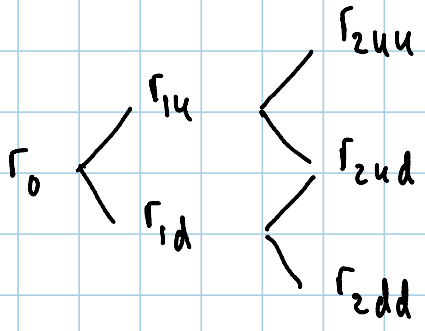
$$P_0(2) = \frac{1}{1+r}\left(q_1 P_{1u}(2) + (1-q_1) P_{1d}(2) \right)$$

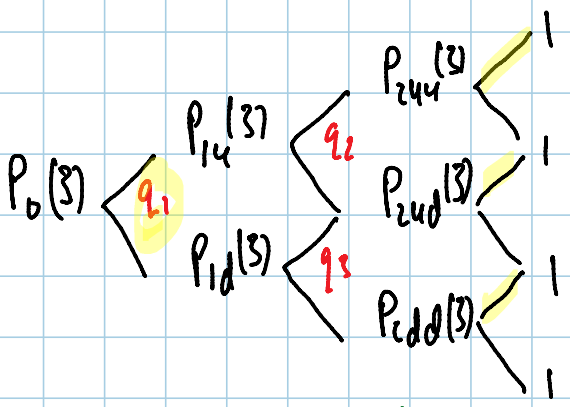




$$S_d < (1+r)S < S_u$$

$\Leftrightarrow$ $\nexists$ arb.





$\rightarrow \frac{1}{1+"r"}$

$P_0(3) = f(q_2, q_3)$

$\Rightarrow$ no unique Q!

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Ho-Lee Model

$$r_n = r_{n-1} + \theta_{n-1}\Delta t + \sigma\sqrt{\Delta t}\, x_n$$

x_n: ± 1 Bernoulli iid, $\mathbb{Q}(x_1 = \pm 1) = 1/2$

$$r_0 \begin{cases} q = 1/2 & r_0 + \theta_0\Delta t + \sigma\sqrt{\Delta t} = r_{1u} \\ & r_0 + \theta_0\Delta t - \sigma\sqrt{\Delta t} = r_{1d} \end{cases}$$

find $\theta_0, \theta_1, \theta_2, \ldots$ to match bond prices.

$$P_0(1) = e^{-r_0\Delta t} \Rightarrow r_0$$

$$P_0(2) = e^{-r_0\Delta t}\left[\tfrac{1}{2}e^{-r_{1u}\Delta t} + \tfrac{1}{2}e^{-r_{1d}\Delta t}\right]$$

$\Rightarrow \theta_0$

θ_1

$$P_0(3) \begin{cases} 1/2 \begin{cases} 1/2 & P_{2uu}(3) = e^{-r_{2uu}\Delta t} \\ & P_{2ud}(3) = e^{-r_{2ud}\Delta t} \end{cases} \\ 1/2 \begin{cases} 1/2 & \\ & \end{cases} \end{cases} \Rightarrow \theta_1$$

$$P_{2dd}(3) = e^{-r_{2dd}\,\Delta t}$$

r_2

$$P_0(T) = e^{-y(T) \times T}$$

$y(T)$ — yield