Noninformative priors

- Welch and Peers (1963) established that, for $\theta \in R$, if

$$r(Y,\theta) = \int^{\theta} \pi(t|Y)dt$$

is to satisfy the property that

$$\text{pr}\{r(Y,\theta) < \alpha|\theta\} = \alpha + O(n^{-1}),$$

then we need to choose $\pi(\theta) \propto \{i_1(\theta)\}^{1/2}$.

- Peers (1965) considered $\theta \in R^m$ and posterior marginals. Let

$$r(Y,\theta_1) = \int^{\theta_1} \int \dots \int \pi(t|Y)dt_p \dots dt_2 dt_1$$

and

$$z(Y,\theta_1) = \Phi^{-1}\{r(Y,\theta_1)\}.$$

Then

$$E(e^{tZ}) \doteq \exp\left[\frac{1}{2}t^2 + \frac{t}{\sqrt{n}}\left\{i^{1j}(i^{11})^{-1/2}\frac{\partial\psi}{\partial\theta_j} + \frac{\partial}{\partial\theta_j}(i^{1j}(i^{11})^{-1/2}\right\} + O(n^{-1})\right]$$

where $\psi(\theta) = \log\pi(\theta)$, i^{1j} is the first row of $i^{-1}(\theta)$, and $i(\theta)$ is throughout the expected Fisher information in one observation.

- In the special case that $i_{1j} = 0$ for $j = 2, \dots, p$, the $1/\sqrt{n}$ term in this expression is zero if and only if

$$\pi(\theta) \propto i_{11}^{1/2}(\theta)g(\theta_2, \dots, \theta_p)$$

where $g(\cdot)$ is an arbitrary function of $(\theta_2, \dots, \theta_p)$. This prior is often called a Tibshirani prior after Tibshirani (1989).

- Peers' conditions cannot be simultaneously satisfied for each component of θ.
- In both the scalar and vector parameter case, the vanishing of the $1/n$ term in the expansion of $\mathrm{pr}\{r(Y,\theta) < \alpha\}$ depends on the model, and not on the choice of prior. This is discussed in WP63 and in Mukerjee and Dey (1993).
- There are many other definitions of 'noninformative' for priors. See the reference to Ghosh and Mukerjee (1992) in my Valencia paper. Priors of the Welch-Peers form are often called 'matching' priors to distinguish them from these other definitions. In the multiparameter setting, there are also ther definitions of matching, that are not equivalent to the Peers definition.
- A noninformative prior getting a lot of attention recently is the reference prior of Berger and Bernardo (1992ab). It attempts to maximize the Kullback-Liebler distance between the prior and the posterior.