Laplace approximation of integrals

1.

$$\int_a^b e^{-ng(y)}dy = e^{-ng(\tilde{y})}\sqrt{\frac{2\pi}{n}}\{g''(\tilde{y})\}^{-1/2}\{1+\frac{5\tilde{\rho}_3^2-3\tilde{\rho}_4}{24n}+O(n^{-2})\}$$

where $g'(\tilde{y})=0$, $g''(\tilde{y})>0$, $\tilde{\rho}_3=g'''(\tilde{y})/\{g''(\tilde{y})\}^{3/2}$, $\tilde{\rho}_4=g^{IV}(\tilde{y})/\{g''(\tilde{y})\}^2$, and we assume here and in the following that the function g has a unique non-zero minimum in the interval (a,b).

2.

$$\int h(y)e^{-ng(y)} = h(\tilde{y})e^{-ng(\tilde{y})}\sqrt{\frac{2\pi}{n}}\{g''(\tilde{y})\}^{-1/2}\{1+\frac{5\tilde{\rho}_3^2-3\tilde{\rho}_4}{24n}+\frac{h''(\tilde{y})}{2g''(\tilde{y})h(\tilde{y})n}-\frac{\tilde{\rho}_3h'(\tilde{y})/h(\tilde{y})}{2\{g''(\tilde{y})\}^{1/2}n}+O(n^{-2})\}$$

for which we need to assume that $h(\tilde{y})\neq 0$.

3.

$$\begin{aligned}\int e^{-n\{g(y)-\frac{1}{n}\log h(y)\}}dy \quad &= \textstyle\int e^{-nq_n(y)}dy \quad \text{, say,}\\ &= e^{-ng(y^*)}h(y^*)\sqrt{\tfrac{2\pi}{n}}\{q_n''(y^*)\}^{-1/2}\{1+\tfrac{5\rho_3^{*2}-3\rho_4^*}{24n}+O(n^{-2})\}\end{aligned}$$

where $q_n'(y^*)=0$, $\rho_j^*=q^{(j)}(y^*)/\{q''(y^*)\}^{j/2}$.

4.

$$\begin{aligned}\int_{R^d} h(y)e^{-ng(y)}dy \quad &= e^{-ng(\tilde{y})}h(\tilde{y})\left(\sqrt{\tfrac{2\pi}{n}}\right)^d|g''(\tilde{y})|^{-1/2}\{1+O(n^{-1})\}\\ &= e^{-ng(y^*)}h(y^*)\left(\sqrt{\tfrac{2\pi}{n}}\right)^d|g''(y^*)|^{-1/2}\{1+O(n^{-1})\}\end{aligned}$$