First order asymptotic theory for likelihood based quantities

As usual we assume that we have a random vector $Y = (Y_1, \ldots, Y_n)$, where $Y_1, \ldots, Y_n$ are independent, identically distributed, from a density function $f(y;\theta)$ where $\theta \in R^p$.

- $p = 1$
  - $\frac{1}{\sqrt{n}} U(\theta)/i^{1/2}(\theta) \xrightarrow{d} N(0,1)$ by the central limit theorem
  - $\sqrt{n}(\hat{\theta} - \theta) i^{1/2}(\theta) = \frac{1}{\sqrt{n}} \frac{U(\theta)}{i^{1/2}(\theta)} \{1 + o_p(1)\}$, where $\hat{\theta}$ satisfies $\ell(\hat{\theta}; Y) = 0$
    - $\sqrt{n}(\hat{\theta} - \theta) i^{1/2}(\theta) \xrightarrow{d} N(0,1)$
  - $w(\theta) = (\hat{\theta} - \theta)^2 n i(\theta) \{1 + o_p(1)\}$
    - $w(\theta) \xrightarrow{d} \chi^2_1$
- $p > 1$
  - $\{U(\theta)\}^T \{n i(\theta)\}^{-1} \{U(\theta)\} \xrightarrow{d} \chi^2_p$
  - $(\hat{\theta} - \theta)^T n i(\theta) (\hat{\theta} - \theta) \xrightarrow{d} \chi^2_p$
  - $w(\theta) \xrightarrow{d} \chi^2_p$
- $\theta = (\psi, \lambda) : \psi \in R^q$
  - $\{U_\psi(\psi, \hat{\lambda}_\psi)\}^T n^{-1} i^{\psi\psi}(\psi, \hat{\lambda}_\psi) \{U_\psi(\psi, \hat{\lambda}_\psi)\} \xrightarrow{d} \chi^2_q$
  - $(\hat{\psi} - \psi)^T n \{i^{\psi\psi}(\psi, \hat{\lambda}_\psi)\}^{-1} (\hat{\psi} - \psi) \xrightarrow{d} \chi^2_q$
  - $2\{\ell(\hat{\psi}, \hat{\lambda}) - \ell(\psi, \hat{\lambda}_\psi)\} \xrightarrow{d} \chi^2_q$