STA3000: Asymptotic theory for likelihood

Order notation(Reference BNC 89 Ch.2 or BDR 07 Appendix)

The definitions of O, O_p, o and o_p are as follows:

(i) Given sequences of constants $\{a_n\}$ and $\{b_n\}$ we say that $a_n = o(b_n)$ if $a_n/b_n \to 0$, as $n \to \infty$ and $a_n = O(b_n)$ if $a_n/b_n \to A$ as $n \to \infty$ where $A < \infty$ is a constant not depending on n.

(ii) Given sequences of random variables $\{X_n\}$ we say that $X_n = o_p(a_n)$ if $X_n/a_n \to 0$ in probability, and $X_n = O_p(a_n)$ if X_n/a_n is bounded in probablity. More explicitly for the latter, given $\epsilon > 0$ there exist constants k_ϵ and $n_0 = n_0(\epsilon)$ such that if $n > n_0$ $\Pr(|X_n| < a_n k_\epsilon) > 1 - \epsilon$.

The following rules for O, o, O_p, and o_p can be verified from their definitions:

1. $O(n^{-a})O(n^{-b}) = O(n^{-a-b})$
2. $O_p(n^{-a})O_p(n^{-b}) = O_p(n^{-a-b})$
3. $O_p(n^{-a})O(n^{-b}) = O_p(n^{-a-b})$
4. $O(n^{-a})o(n^{-b}) = o(n^{-a-b})$
5. $O_p(n^{-a})o(n^{-b}) = o_p(n^{-a-b})$
6. $o_p(n^{-a})o_p(n^{-b}) = o_p(n^{-a-b})$

References

[BNC89] Barndorff-Nielsen & Cox (1989). *Asymptotic Techniques.*
[BDR] Brazzale, A.R., Davison, A.C. & Reid, N. (2007). *Applied Asymptotics.*