

Solutions to Exercises - Lecture 13

Ex III.1.1 Suppose X is a r.v. that takes only many finitely values say c_1, \dots, c_m . Then let $C_i = X^{-1}(\{c_i\}) \in \mathcal{A}$ since $\{c_i\} \in \mathcal{B}'$ and X is a r.v. Therefore $X = \sum_{i=1}^m c_i I_{C_i}$ and so takes the form of a simple function.

Ex III.1.2 Suppose $X_1 = \sum_{i=1}^m b_i I_{B_i}$ and $X_2 = \sum_{i=1}^n c_i I_{C_i}$ then

$$a_0 + a_1 X_1 + a_2 X_2 = a_0 I_{\Omega} + \sum_{i=1}^m a_1 b_i I_{B_i} + \sum_{i=1}^n a_2 c_i I_{C_i}$$

and so $a_0 + a_1 X_1 + a_2 X_2$ is a simple function and so by definition

$$\begin{aligned} E(a_0 + a_1 X_1 + a_2 X_2) &= a_0 P(\Omega) + \sum_{i=1}^m a_1 b_i P(B_i) + \sum_{i=1}^n a_2 c_i P(C_i) \\ &= a_0 + a_1 \sum_{i=1}^m b_i P(B_i) + a_2 \sum_{i=1}^n c_i P(C_i) \\ &= a_0 + a_1 E(X_1) + a_2 E(X_2). \end{aligned}$$