Exercise 1.5.3

Proof: We need to prove that P is countably additive. So let $A_1, A_2, \ldots \in \mathcal{A}$ be mutually disjoint. Now $\bigcup_{i=1}^{n} A_i$ is monotone increasing and so by the continuity of P we have that $P(\bigcup_{i=1}^{n} A_i) \to P(\bigcup_{i=1}^{\infty} A_i)$. But since P is finitely additive we have that $P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$.

Therefore
$$\sum_{i=1}^{\infty} P(A_i) = \lim_{n\to\infty} \sum_{i=1}^{n} P(A_i)$$
$$= \lim_{n\to\infty} P(\bigcup_{i=1}^{n} A_i) = P(\bigcup_{i=1}^{\infty} A_i).$$