Review – Operations with Sets

For any two events (sets), E and F we can define:

1) $E \cup F$ = the set containing those points that are in either E **or** F (i.e. points that are in E, or F or in both).

$E \cup F$ is called **the union** between sets E and F.

2) $E \cap F$ = the set containing only those points that are in E **and** in F.

$E \cap F$ is called **the intersection** between sets E and F.

3) E^c = the set containing those points which are not in E (but are in the sample space S).

E^c is called **the complement** of the set E.

4) $\emptyset$ denotes the empty set. For any event E

$$E \cap \emptyset = \emptyset$$

$$E \cup \emptyset = E$$

Review - Operations with Sets

- $[E^c]^c = E$
- $A = (A \cap B) \cup (A \cap B^c)$.
- **De Morgan's First Law**

$$(\cup_{i=1}^{n} E_i)^c = \cap_{i=1}^{n} E_i^c$$

- **De Morgan's Second Law**

$$(\cap_{i=1}^{n} E_i)^c = \cup_{i=1}^{n} E_i^c$$

- **Distributive Laws**

1.

$$F \cap (\cup_{i=1}^{n} E_i) = \cup_{i=1}^{n} (F \cap E_i)$$

2.

$$F \cup (\cap_{i=1}^{n} E_i) = \cap_{i=1}^{n} (F \cup E_i)$$

Review - New Notations

- '*E happens but F doesn't*' is denoted $E - F$
- *Symmetrical Difference* is denoted $E \Delta F$.

$$E \Delta F = (E - F) \cup (F - E)$$

Review - The Case of three events

Say there are three events: A,B, and C

- Exactly one of them is true

$$(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$$

- Exactly two are true

$$(A \cap B \cap C^c) \cup (A^c \cap B \cap C) \cup (A \cap B^c \cap C)$$

- All three are true

$$A \cap B \cap C$$

- None of them is true

$$(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$$

- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
- Generalization:

$$P(\cup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i) - \sum_{i_1 < i_2} P(E_{i_1} \cap E_{i_2})$$

$$+ \sum_{i_1 < i_2 < i_3} P(E_{i_1} \cap E_{i_2} \cap E_{i_3}) + \ldots$$

$$\ldots (-1)^{r+1} \sum_{i_1 < i_2 < \ldots < i_r} P(E_{i_1} \cap E_{i_2} \cap \ldots \cap E_{i_r}) + \ldots$$

$$\ldots (-1)^{n+1} P(E_1 \cap \ldots \cap E_n)$$

Example:

$$P(E_1 \cup E_2 \cup E_3 \cup E_4) = P(E_1) + P(E_2) + P(E_3) + P(E_4) -$$

$$-P(E_1 \cap E_2) - P(E_1 \cap E_3) - P(E_1 \cap E_4) -$$

$$-P(E_2 \cap E_3) - P(E_2 \cap E_4) - P(E_3 \cap E_4) +$$

$$+P(E_1 \cap E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_4) +$$

$$+P(E_1 \cap E_3 \cap E_4) + P(E_2 \cap E_3 \cap E_4) -$$

$$-P(E_1 \cap E_2 \cap E_3 \cap E_4)$$