STA 257 – Summer 2003

Multivariate Calculus Practice Problems

Recommended preparation for tutorial on Thursday, June 5

In the following questions, unless stated otherwise, evaluate the integral.

1. \( \int_0^2 x^2 y^3 \, dy \) \hspace{1cm} [Ans: 4x^2]

2. \( \int_0^1 \int_0^2 x \sqrt{y} \, dx \, dy \) \hspace{1cm} [Ans: \frac{32}{3}]

3. \( \int_{-1}^1 \int_0^1 (x^3 y^3 + 3xy^2) \, dy \, dx \) \hspace{1cm} [Ans: 0]
   Is there a quick way of doing this question?

4. Sketch the region of integration and change the order of integration for the integral
   \[
   \int_0^1 \int_0^{x^2} f(x, y) \, dy \, dx
   \]

5. \( \int_0^1 \int_0^y x \, dx \, dy \) \hspace{1cm} [Ans: \frac{1}{6}]

6. Repeat the previous question but change the order of integration.

7. Find
   \[
   \int \int_D xy \, dA
   \]
   where \( D \) is the region bounded by the \( x \)-axis and the lines \( x = 2 \) and \( y - 2x = 0 \).
   [Ans: 8]

8. Find
   \[
   \int \int_D x^2 - 2xy \, dA
   \]
   where \( D = \{(x, y) \mid 0 \leq x \leq 1, \sqrt{x} \leq y \leq 2 - x\} \). [Ans: \(-\frac{19}{42}\)]

9. Find
   \[
   \int \int_D e^{x+y} \, dA
   \]
   where \( D \) is the region bounded by the \( x \)-axis and the lines \( x = y \) and \( x = 1 \). [Ans: \(1.5e^2 - e + .5\)]

10. Find
    \[
    \int \int_D e^x \, dA
    \]
    where \( D \) is the triangular region with vertices \((0,0), (2,4), \) and \((6,0)\). [Ans: \(e^6 - 3e^2 + 2\)]