STA 257 – Fall 2002

Practice Problems 6
Recommended preparation for quiz to be held in tutorial on Wednesday, November 6

Sections from Schaeffer covered October 21 through October 30: 5.4, 5.7, 3.2 (Chebyshev’s Inequality), 7.2

Questions from the textbook (Schaeffer):

1. From Section 5.4: 5.19
2. From Section 5.7: 5.43
3. From Chapter 5 Supplementary Exercises: 5.57
4. From Section 3.2: 3.19
5. From Section 7.3 (although these questions are on material from Section 7.2): 7.1, 7.4

Additional questions:

6. Find an expression for the variance of $XY$ if $X$ and $Y$ are independent. [Ans: $V(XY) = E(X^2) \cdot E(Y^2) - (EX)^2 \cdot (EY)^2$]

7. Verify the following properties of covariance. (Here $X$, $Y$, and $Z$ are random variables and $a$, $b$, $c$, and $d$ are constants.)
   (a) $\text{Cov}(aX + b, cY + d) = ac\text{Cov}(X, Y)$
   (b) $\text{Cov}(X + Y, Z) = \text{Cov}(X, Z) + \text{Cov}(Y, Z)$
   (c) $\text{Cov}(Y, X) = \text{Cov}(X, Y)$

8. Let $X$ and $Y$ have the joint distribution shown in the table. Find their covariance and their correlation coefficient.

<table>
<thead>
<tr>
<th>$p(x, y)$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>.12</td>
<td>.08</td>
<td>.11</td>
</tr>
<tr>
<td>$y$</td>
<td>.18</td>
<td>.14</td>
<td>.07</td>
</tr>
<tr>
<td>$z$</td>
<td>.17</td>
<td>.05</td>
<td>.08</td>
</tr>
</tbody>
</table>

[Ans: Cov = -.0821, $\rho = -.1269$]

9. Let $X$ and $Y$ have the following joint density function:

$$f(x, y) = \frac{6}{5}(x^2 + y) \text{ for } 0 < x < 1 \text{ and } 0 < y < 1 \text{ (and 0 otherwise)}$$

Find the covariance and correlation coefficient. [Ans: Cov = $-1/100$, $\rho = -1.30558$]

10. Show that for any two random variables $X$ and $Y$, $\text{Cov}(X + Y, X - Y) = VX - VY$. 

1
11. Let $Y_1$, $Y_2$, $Y_3$, ... be independent and identically distributed random variables, with $EY_j = \mu$ and $VY_j = \sigma^2$. Let $Y = Y_1 + Y_2 + Y_3 + \cdots + Y_X$, where $X$ is a positive integer-valued random variable. Show that

$$EY = \mu EX$$

and

$$VY = \mu^2 VX + \sigma^2 EX$$

(provided, of course, that $EX$ and $VX$ exist).

12. Let $X$ have an exponential distribution with mean $1/\lambda$. Given that $X = x$, let $Y$ conditionally have the normal distribution with expected value $\mu X$ and variance $\sigma^2 X^2$. Find $EY$ and $VY$. [Ans: $EY = \mu \lambda$, $VY = (\mu^2 + 2\sigma^2)/\lambda^2$]

13. Suppose $A$ is an event associated with some chance experiment and $P(A) = .35$. Suppose the experiment is repeated 45 times, and on the $i$th repetition $X_i$ equals 1 if $A$ occurs and 0 if not.

(a) What is the distribution of $S_{45} = \sum_{i=1}^{45} X_i$?
(b) What are $E\overline{X}_{45}$ and $V\overline{X}_{45}$?
(c) What does Chebyshev’s inequality say about the probability that $\overline{X}_{45}$ is not between .3 and .4?
(d) How would your answers change if the experiment was repeated 4500 times instead of 45?

[Ans: (a) Binomial with $n = 45$ and $p = .35$ (b) .35, .0050506 (c) Less than or equal to 2.02]

Relevant sections in Schaeffer for lectures during the week of November 4: 6.1, 6.2, 6.3, 6.4