Exercise 4.40 from *OpenIntro Statistics*
A manufacturer of compact fluorescent light bulbs advertises that the distribution of the lifespans of these light bulbs is nearly normal with a mean of 9,000 hours and a standard deviation of 1,000 hours.

a) What is the probability that a randomly chosen light bulb lasts more than 10,500 hours?

b) Describe the distribution of the mean lifespan of 15 light bulbs.

c) What is probability that the mean lifespan of 15 randomly chosen light bulbs is more than 10,500 hours?

d) Sketch the two distributions (population and sampling) on the same scale.

e) Could you estimate the probabilities from parts a) and c) if the lifespans of light bulbs had a skewed distribution?
Exercise 4.40 from *OpenIntro Statistics*

A manufacturer of compact fluorescent light bulbs advertises that the distribution of the lifespans of these light bulbs is nearly normal with a mean of 9,000 hours and a standard deviation of 1,000 hours.

a) What is the probability that a randomly chosen light bulb lasts more than 10,500 hours?

\[ X = \text{lifespan of a light bulb} \]

\[
P(X > 10,500) = P\left(Z > \frac{10,500 - 9,000}{1,000}\right) \quad \text{where} \quad Z \sim N(0, 1)
\]

\[
= P(Z > 1.5)
\]

\[
= 1 - 0.9332 = 0.0668
\]
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b) Describe the distribution of the mean lifespan of 15 light bulbs.

\[ X_1, X_2, \ldots, X_{15} \sim N(9000, 1000^2) \]

\[ \bar{X} \sim N \left( 9000, \frac{1000^2}{15} \right) \]

\[ \sigma \sim N \left( 9000, \frac{1000}{\sqrt{15}} \right) \]
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with a mean of 9,000 hours and a standard deviation of 1,000 hours.

c) What is the probability that the mean lifespan of 15 randomly chosen
light bulbs is more than 10,500 hours?

\[
P\left(\bar{X} > 10500 \right) = P \left( Z > \frac{10500 - 9000}{1000/\sqrt{15}} \right)
\]

\[
= P \left( Z > 5.8 \right)
\]

is tiny.
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e) Could you estimate the probabilities from parts a) and c) if the lifespans of light bulbs had a skewed distribution?
Count of the number of successes, $X$

$X \sim \text{Binomial} \left( n, p \right)$

Number of trials

Prob. of success

Mean $\mathbb{E}(X) = np$,

$\text{Var}(X) = np(1-p)$

If $n$ is large

$x$ has approx. Normal dist.

Proportion, $\hat{p} = \frac{X}{n}$

If $n$ is large,

$\hat{p} \sim N \left( p, \frac{p(1-p)}{n} \right)$