Two-way ANOVA

Model:

\[ Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} \]

Assumptions:

\[ \epsilon_{ijk} \text{ i.i.d. } N(0, \sigma^2) \]

- all observations independent
- all groups have same variance
- data are normally distributed
Examples of interaction (or not)

Interaction plot & Response curve

\[ X_1 \text{ has 2 levels} \]
\[ X_2 \text{ has 2 levels} \]

No interaction
- how \( X_1 \) affects \( Y \) is same when \( X_2 = 1 \) and \( X_2 = 2 \)
- \( X_1 \) has parallel lines

With interaction:
- \( Y \) for \( X_1 = 2 \) is \( > \) \( X_1 = 1 \)
  - but opposite when \( X_2 = 2 \)
If response curves are parallel, $X_1$ and $X_2$ do not interact, relationship between $Y$ and $(X_1, X_2)$ is simpler.

Could fit the model:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \varepsilon_{ijk}$$

an additive model.

If there is a statistically significant interaction, no need to test whether there is a "main effect" of $X_1$ or $X_2$. Their effects are tied up with the level of the other factor.
Tests for interactions and main effects are tested via F-tests via decomposition of total SS as in 1-way ANOVA.

Interaction plots

3 levels of $X_1$
2 of $X_2$

1 $X_1, X_2$ interact

1 2 3 $X_1$

$X_2 = 2$
$X_2 = -1$
1 2 3 $X_1$
(2) No interaction, $X_1$ has an effect, 
$X_2$ doesn't.

![Graph showing mean of $Y$ with $X_1$ for $X_2 = 1$ and $X_2 = 2$.]

(3) No interaction, $X_1$ and $X_2$ both have an affect on $Y$.

![Graph showing mean of $Y$ with $X_1$ for $X_2 = 1$ and $X_2 = 2$.]

(4) No interaction, no effect of $X_1$,
$X_2$ affects $Y$.

![Graph showing mean of $Y$ with $X_1$ for $X_2 = 1$ and $X_2 = 2$.]
Randomized Block Design

- Two factor analysis of variance
  - One of interest
  - Other a source of variation
  - Can control for "block"

E.g. Interests in comparing effects of fertilizers on crop yield.

- Have 6 fields
  - Fields are relatively homogeneous

- Divide fields into 3 pots and randomly assign fertilizer to plot within each field
  - Fields are blocks

- Generally not interested in block-treatment interaction.