Model: \[ Y_{ij} = \mu + \alpha_i + \epsilon_{ij} \]

- \( \mu \) is the population mean
- \( \alpha_i \) represents the fixed effect of the \( i \)th group
- \( \epsilon_{ij} \) is the random error

\[ i = 1, \ldots, a \]
\[ j = 1, \ldots, n_i \]
\[ N = n_1 + n_2 + \ldots + n_a \]

Test:
- \( H_0 \): each group has same mean
- \( H_a \): at least one pair of groups has different means

\[ H_0: \alpha_1 = \alpha_2 = \ldots = \alpha_a = 0 \]
\[ H_a: \text{at least one of these is } \neq 0 \]
### ANOVA Table

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groups</td>
<td>a-1</td>
<td>SSTr</td>
<td>MSr</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>N-a</td>
<td>SSE</td>
<td>MSE</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>N-1</td>
<td>SSTb</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Mean Square**

\[
MSr = \frac{SSTr}{a-1} \\
MSE = \frac{SSE}{N-a}
\]

**Ideal:** Compare variability of values within groups to variability between groups.

- If similar, no difference.

**It can be shown,**

\[
E(\text{SSTr}) = \sum_{i=1}^{a} n_i d_i^2 + (a-1) \sigma^2
\]

\[
E(\text{MSTr}) = \frac{\sum_{i=1}^{a} n_i d_i^2}{a-1}
\]

**Variable between treatments**

\[
E(\text{SSE}) = (N-a) \sigma^2
\]

\[
\Rightarrow E(\text{MSE}) = \sigma^2
\]

MSE is an unbiased estimate of VAR(\(E_{ij}\)).
Can show (exercise):

\[ MS_E = \frac{\sum_{i=1}^{a} (n_i-1) s_i^2}{\sum_{i=1}^{a} (n_i-1)} \]

where \( s_i^2 = \frac{\sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_i)^2}{n_i-1} \) (variance of observations in i-th group)

pooled variance

\[ S_p^2 = MS_E \]

Fact: Under \( H_0: \alpha_1 = \alpha_2 = \ldots = \alpha_a = 0 \)

\[ F_{obs} = \frac{MS_{Tr}}{MS_E} \] has an F distribution on \( a-1 \) and \( N-a \) df

\( F \) distribution has 2 parameters: numerator and denominator df.
If $H_0$ is true
$E(MS_{TR}) = E(MS_E) = \sigma^2$
So $F_{obs}$ should be $\sim 1$

If $H_0$ is false, $F_{obs}$ should be $> 1$
So reject $H_0$ for large values of $F_{obs}$

(only calculate p-value in right tail, even though $H_a$ is 2-sided)

Sample Example

Comparison of all 7 judges

ANOVA Table:

<table>
<thead>
<tr>
<th>JUDGE</th>
<th>DF</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F stat</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>127.08</td>
<td>21.18</td>
<td>4.78</td>
<td>0.05</td>
</tr>
<tr>
<td>Residuals</td>
<td>39</td>
<td>1864.45</td>
<td>47.81</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
p-value < 0.05

$F_{8,39} < 0.05$

\[ G = 7184 \]

PCW.aov & coef

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coef</th>
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</thead>
<tbody>
<tr>
<td>Intercept</td>
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</tr>
<tr>
<td>JUDGEA</td>
<td>-0.503</td>
</tr>
<tr>
<td>C</td>
<td>-6.02</td>
</tr>
<tr>
<td>D</td>
<td>-7.12</td>
</tr>
<tr>
<td>E</td>
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</tr>
<tr>
<td>F</td>
<td>-7.35</td>
</tr>
<tr>
<td>Slopes</td>
<td>-19.5</td>
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</tbody>
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