Sample of 9 components:
average temperature of 41.08°C
Assume temperatures follow a Normal distribution with S.D. 1.5°C
Test: $H_0: \mu = 40$
$H_a: \mu > 40$
Calculation:

$Z_{obs} = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

$\bar{X}_{obs} = 41.08 - 40 = 1.08$

$Z_{calc} = \frac{1.08}{1.5 / \sqrt{9}} = 2.16$

If the true test statistic has $N(0,1)$ distribution

$p$-value = $P(Z > 2.16)$

$p$-value = 0.0154

We have some evidence against $H_0$ (not strong)

Assuming $H_0$ is true, chance of seeing what we got or even more extreme difference from $H_0$

is only 3%
as so we saw something fairly rare.

Interpreting $p$-values

$p$-value > 0.1
large p-value, no evidence against $H_0$

0.01 < p-value < 0.1
weak evidence against $H_0$

p-value > 0.01
some evidence against $H_0$

p-value < 0.01
strong evidence against $H_0$

When $H_0$ is not rejected: (could be a Type II error)

maybe test was not powerful enough to find a difference between observed average and mean when claimed in $H_0$

Power = 1 - $\beta$
prob. of Type II error

Consider upper - tailed test
$H_0: \mu = \mu_0, H_a: \mu > \mu_0$

with significance level $\alpha$

Reject $H_0$ if

$Z_{calc} = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} > Z_{\alpha}$

So won't reject if $\bar{X} \leq \mu_0$

Suppose $\mu'$ is a value for $\mu$ that exceeds $\mu_0$

would reject $H_0$ if true mean is $\mu'$
\[ \beta(\mu^*) = \Phi \left( \frac{\bar{X} - \mu^*}{\sigma/\sqrt{n}} \right) \]

If \( \mu = \mu^* \), then \( \bar{X} \sim N(\mu^*, \sigma^2/n) \)

\[ \beta(\mu^*) = \Phi \left( \frac{\bar{X} - \mu^*}{\sigma/\sqrt{n}} \right) \]

\[ \text{Power} (\mu^*) = 1 - \Phi \left( \frac{\bar{X} - \mu^*}{\sigma/\sqrt{n}} \right) \]

To increase power:

- Increase \( \alpha \)
- Decrease \( \sigma \)
- Increase \( n \)
- \( \mu^* \) further away from \( \mu_0 \)

If the alternative was \( H_0: \mu \leq \mu_0 \)

\[ \mu^* \text{ value less than } \mu_0 \]

\[ \beta(\mu^*) = 1 - \Phi \left( -\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \right) \]

Power (\( \mu^* \)) = \[ \Phi \left( \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \right) \]

To increase power:

- Increase \( \sigma \)
- Decrease \( n \)

### Case I: Large Sample Tests

When \( \sigma^2 \) is known, still testing \( \mu \)

\[ Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \]

\[ s = \sqrt{\frac{1}{n-1} \sum (X_i - \bar{X})^2} \]

Has approx a standard normal distribution under \( H_0 \) (\( \mu = \mu_0 \)) for large \( n \)

(see \( n \approx 40 \))

### Case III: Suitable for any \( n \)

Use distribution of \( \bar{X} \) approx normal

Test statistic

\[ T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \]

\( S \) has approx. a t-distribution with \( n-1 \) df.

Proceed as previous except replace \( Z \) with \( t_{(n-1)} \) or \( t_{(n-1)} \) with \( t_{(n-1)} \).
Example: Marks data
- Assume sample of size 20 from a large population of students.
  Do we evidence that the mean mark is more than a passing grade?
  \[ H_0: \mu \leq 50 \]
  \[ H_a: \mu > 50 \]

  Test statistic:
  \[
t_{obs} = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}
  \]
  \[
  t_{obs} = \frac{54.95 - 50}{14.1 / \sqrt{20}} = 1.57
  \]

  Under \( H_0 \), \( t \approx t_{19} \)

  From tables
  \[ t_{19} \text{ at } 0.05 < p \text{ value } < 0.1 \]

  We have weak evidence that the mean mark is 55.

Example: Test of proportion
- 46.7% of 500 Canadians surveyed support gay marriage. Would a referendum pass?

  Let \( p \) be the proportion of Canadians who support gay marriage.

  \[ p = 0.47 \]

  If \( X \) is the number of supporters in sample of size 500

  \[ X \sim \text{Binomial}(500, p) \]

  \[ E(X) = np \]
  \[ \text{Var}(X) = np(1-p) \]

  \[ \hat{p} = \frac{X}{500} \]
  \[ \text{Var} \hat{p} = \frac{p(1-p)}{500} \]

  By CLT, \( \hat{p} \) has approx. a normal distribution.

  Test:
  \[ H_0: p \leq 0.5 \]
  \[ H_a: p > 0.5 \]