STA 410/2102, Spring 2004 — Assignment #2

Due at **start** of class on March 16. Worth 20% of the final mark.

Note that this assignment is to be done by each student individually. You may discuss it in general terms with other students, but the work you hand in should be your own.

In this assignment, you will use Newton-Raphson iteration to compute maximum likelihood estimates for the parameters of a "wrapped normal" distribution for circular data, in which each data point is an angle. Angles will be measured in radians, ranging from 0 to 2π.

One can visualize the wrapped normal distribution with parameters μ and σ by imagining that data points are generated from the normal distribution with mean μ and standard deviation σ, and these data points are then "wrapped" around a circle. This is done by transforming data point x to a wrapped data point x' such that $x' = x + 2\pi i$ for some integer i, chosen so that $x' \in [0, 2\pi)$.

The probability density function for a wrapped normal distribution is as follows:

$$f(x) \;=\; \sum_{i=-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu-2\pi i)^2\right)$$

The sum accounts for all the ways that wrapping could produce the value x.

Problem 1. Write an R function to evaluate the wrapped normal density for a vector of x values, using given single values for μ and σ. You will need to come up with some way of deciding how many terms in the infinite sum above need to be evaluated in order to obtain an accurate approximate of the density.

Problem 2. Write an R function to find a maximum likelihood estimate for μ, with σ fixed. This function should take as arguments a vector of circular data points, x, a specified fixed value for σ, and an initial guess for μ. It should return a possible maximum likelihood estimate for μ, as obtained by using Newton-Raphson iteration to find a point where the derivative of the log likelihood with respect to μ is zero. You may write your function to do some fixed number of Newton-Raphson iterations, chosen so that it seems sufficient to reach convergence, if the Newton-Raphson iteration is converging at all.

By trying various initial guesses for μ, you should investigate whether Newton-Raphson iteration always converges for this problem, whether the values found are actually local maxima of the likelihood, and whether there is more than one local maximum. Discuss your results.

Problem 3. Write an R function to find a maximum likelihood estimate for both μ and σ (ie, with neither fixed), using Newton-Raphson iteration. Rather than working in terms of σ itself, you should work in terms of $\rho = \log(\sigma)$, which has an unbounded range. Your function should take as arguments a vector of circular data points, x, and initial guesses for μ and ρ. It should return a list with elements called `mu` and `rho` containing the maximum likelihood estimates, as well as an element called `cov`, which is an estimate of the covariance matrix for the estimates of μ and ρ, obtained from the observed information matrix. You may fix the number of iterations, as for Problem 2. You should carry out investigations and discuss the results as for Problem 2

You should first test your functions for Problem 2 and Problem 3 on various data sets you devise yourself. On March 11, I will provide some data sets on which you should run these functions. You should hand in a listing of your functions, the results of running your functions on these test data sets, as well as any other data sets that you think show something interesting, and your discussion of these results.