Example: Given P(A) = 0.25, P(B) = 0.80, and P(A or B) = 0.65. Are A and B disjoint? Are they independent?

$P(A \text{ or } B) = P(A) + P(B) = 0.25 + 0.80$

$= 1.05 > 1$

$\Rightarrow$ A, B are not disjoint

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\Rightarrow P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$

$= 0.25 + 0.80 - 0.65 = 0.40$

$P(A) \cdot P(B) = 0.25 \cdot 0.80 = 0.20$

So, $P(A \text{ and } B) \neq P(A) P(B)$

$\Rightarrow$ A, B are not independent.

Example: There are 10 balls in the box: 4 blue ones, 3 red ones, and 3 green ones. You select a ball at random, note its colour, then put it back (replace it). Then you take a ball second time.

$P(b) = 4/10$

(a) What is the probability that both balls you selected are blue?

$P(b \text{ and } b) = P(b)\,P(b) = \frac{4}{10} \cdot \frac{4}{10} = \frac{16}{100}$

(b) What is the probability that neither is blue?

$P(\text{not } b \text{ and not } b) = \frac{6}{10} \cdot \frac{6}{10} = \frac{36}{100}$

(c) What is the probability that one is blue and another is green?

$P(b \text{ and } g) + P(g \text{ and } b) = \frac{4}{10}\frac{3}{10} + \frac{3}{10}\cdot\frac{4}{10} = \frac{24}{100}$

(d) What is the probability that the balls are of the same colour?

$P(b \text{ and } b) + P(g \text{ and } g) + P(r \text{ and } r)$

$= \frac{4}{10}\cdot\frac{4}{10} + \frac{3}{10}\cdot\frac{3}{10} + \frac{3}{10}\cdot\frac{3}{10} = \frac{34}{100}$

Example: In Exton School, 40% of the girls like music and 12% of the girls like music and dance. What percent of those that like music also like dance?

$A = \{\text{music}\}$

$B = \{\text{dance}\}$

$P(A) = 0.4$

$P(A \text{ and } B) = 0.12$

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.12}{0.4}$$

$$= 0.30$$

$\Rightarrow 30\%$

Ch. 14, # 19

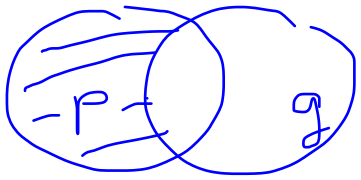
Real estate ads suggest that 64% of homes for sale have garages, 21% have swimming pools, and 17% have both features. What is the probability that a home for sale has

(a) A pool or garage?

$$P(p \text{ or } g) = P(p) + P(g) - P(p \text{ and } g)$$
$$= 0.21 + 0.64 - 0.17 = 0.68$$

(b) Neither a pool nor a garage?

$$P(\text{neither}) = 1 - P(p \text{ or } g) = 1 - 0.68 = 0.32$$



(c) A pool but no garage?

$$P(p \text{ and } g^c) = P(p) - P(p \text{ and } g)$$
$$= 0.21 - 0.17 = 0.04$$

Ch. 15, # 7:

You roll a fair die three times. What is the probability that

(a) You roll all 6s? $P('6') = 1/6$

$$P(\text{all } '6') = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$$

(b) You roll all odd numbers? $P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

(c) You roll at least one 5?

$$P(\text{at least one } '5') = 1 - P(\text{none})$$

$$= 1 - \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = 1 - \frac{125}{216} = \frac{91}{216}$$