Lecture 3

Probability Distribution

Discrete Case

Definition: A r.v. Y is said to be **discrete** if it assumes only a finite or countable number of distinct values.

$P(Y=y)$ = probability that Y takes value y

Definition: The probability that Y takes on the value y, $P(Y=y)$, is defined as the sum of the probabilities of all points in S that are assigned the value y.

$p(y) = P(Y = y)$ is called a **probability function** for Y.

Definition: The **probability distribution** for Y can be described by a formula, table, or a graph that provides $p(y)$ for all y.

Theorem: For any discrete probability distribution, $p(y)$,

1. $0 \leq p(y) \leq 1$ for all y

2. $\sum_{y:P(Y=y)\neq 0} p(y) = 1$

<u>Example</u>: (#3.6) Five balls (#1, 2, 3, 4, 5) are placed in an urn. Two balls are randomly selected from the five. Find the probability distribution for

(a) the largest of 2 numbers;

(b) the sum of two numbers.

<u>Solution</u>: $S = \{(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)\}$

(a)

$card(S) = \binom{5}{2} = 10$

Y = largest of 2 numbers

$p(2) = \frac{1}{10}$, $p(3) = \frac{2}{10}$, $p(4) = \frac{3}{10}$

$p(5) = 4/10$

Y	2	3	4	5
p	1/10	2/10	3/10	4/10

(b) Y = sum of 2 numbers

Y	3	4	5	6	7	8	9
p	1/10	1/10	2/10	2/10	2/10	1/10	1/10

Expected Values

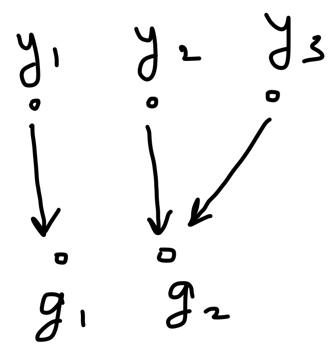
Definition: Let Y be a discrete r.v. with probability function $p(y)$. Then the **expected value** of Y, $E(Y)$, is given by

$$E(Y) = \sum_{y} y p(y)$$

If $p(y)$ is an accurate characteristic of the population distribution, then $E(Y) = \mu$ is the **population mean**.

Theorem: Let Y be a discrete r.v. with probability function $p(y)$ and $g(Y)$ be a real-valued function of Y. Then the expected value of $g(Y)$ is given by

$$E[g(Y)] = \sum_{y} g(y)p(y).$$



Proof: $Y: y_1, \dots, y_n$ $\quad m \le n$

$g(Y): g_1, \dots, g_m$

$$P[g(Y)=g_i] = \sum_{y_j : g(y_j)=g_i} p(y_j) = p^*(g_i)$$

$$E[g(Y)] \overset{def}{=} \sum_{i=1}^{m} g_i p^*(g_i) = \sum_{i=1}^{m} g_i \Big[\sum_{y_j : g(y_j)=g_i} p(y_j)\Big]$$

$$= \sum_{i=1}^{m} \sum g_i p(y_j) = \sum_{j=1}^{n} g(y_j) p(y_j)$$

∎

Definition: Let Y be a r.v. with $E(Y) = \mu$, the **variance** of a r.v. Y is given by

$$Var(Y) = E[(Y-\mu)^2]$$

The **standard deviation** of Y is the positive square root of Var(Y).

σ^2 is population variance, σ is st. dev.

Example: Given a probability distribution for Y:

y	p(y)
0	1/8
1	1/4
2	3/8
3	1/4

$$E(Y) = \sum_{y=0}^{3} y\, p(y)$$

$$= 0 \cdot \frac{1}{8} + 1 \cdot \frac{1}{4} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{4} = 1.75 = \mu$$

$$Var(Y) = E[(Y-1.75)^2]$$

$$= \sum_{y=0}^{3} (y-1.75)^2 p(y)$$

$$= (0-1.75)^2 \cdot \frac{1}{8} + (1-1.75)^2 \cdot \frac{1}{4} + (2-1.75)^2 \cdot \frac{3}{8} + (3-1.75)^2 \cdot \frac{1}{4}$$

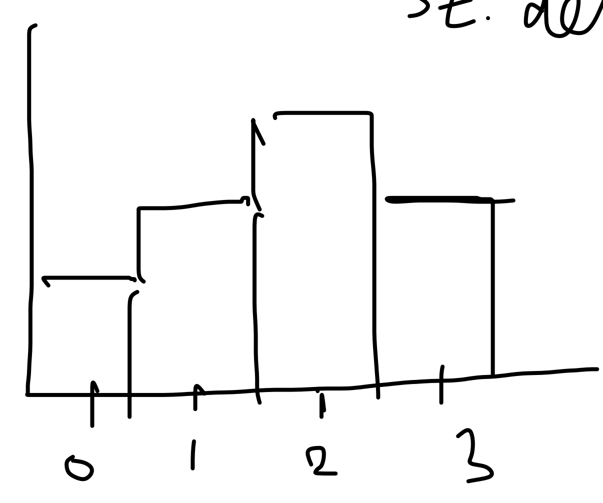
$$= 0.9375$$

$$\text{st. dev} = \sqrt{0.9375} = \underbrace{0.97}_{\sigma}$$

$$(\mu \pm \sigma) = (1.75 \pm 0.97)$$

$\{1,2\}$ are in this interval

$$p(1) + p(2) = \frac{1}{4} + \frac{3}{8} = \frac{5}{8} \approx 68\%$$



Theorem: Let Y be a discrete r.v. with probability function $p(y)$. Then

1. $E(c) = c$, c is a constant;
2. $E[cg(y)] = cE[g(Y)]$, $g(Y)$ is a function of Y;
3. $E[g_1(Y) + \cdots + g_k(Y)] = E[g_1(Y)] + \cdots + E[g_k(Y)]$, $g_1(Y), \dots, g_k(Y)$ are functions of Y.

Proof:

(1) $E(c) = \sum_y c\,p(y) = c \underbrace{\sum_y p(y)}_{=1} = c$

(2) $E(cg(Y)) = \sum c\,g(y)p(y) = c \sum g(y)p(y)$
$= c\,E(g(Y))$

(3) $E[g_1(Y) + \dots + g_k(Y)]$
$= \sum (g_1(y) + \dots + g_k(y))\, p(y)$
$= \sum_y g_1(y)p(y) + \dots + \sum_y g_k(y)p(y)$
$= E(g_1(Y)) + \dots + E(g_k(Y))$

Theorem: $Var(Y) = E[(Y-\mu)^2] = E(Y^2) - \mu^2$,

where $E(Y) = \mu$.

Proof:

$$
\begin{aligned}
Var(Y) &= E[(Y-\mu)^2] \\
&= E[Y^2 - 2Y\mu + \mu^2] \\
&= E(Y^2) - 2\mu \underbrace{E(Y)}_{\mu} + \mu^2 \\
&= E(Y^2) - 2\mu^2 + \mu^2 \\
&= E(Y^2) - \mu^2 \qquad \blacksquare
\end{aligned}
$$

Example: use previous example with $\mu = 1.75$

$$
\begin{aligned}
E(Y^2) &= \sum_y y^2 p(y) \\
&= 0^2 \cdot \frac{1}{8} + 1^2 \cdot \frac{1}{4} + 2^2 \cdot \frac{3}{8} + 3^2 \cdot \frac{1}{4} \\
&= \frac{1}{4} + \frac{3}{2} + \frac{9}{4} = 4
\end{aligned}
$$

$$
\begin{aligned}
Var(Y) &= 4 - 1.75^2 \\
&= 0.9375
\end{aligned}
$$

Binomial Probability Distribution

Definition: A binomial experiment possesses the following properties:

1. It consists of a fixed number, n, of trials;
2. Each trial results in either success, S, or failure, F;
3. p is a probability of successes, $q = 1 - p$ is a probability of failure;
4. The trials are independent;
5. The r.v. of interest, Y, is the number of successes observed during the n trials.

Examples: flipping a coin

rolling a die

Definition: A r.v. Y is said to have a **binomial distribution** based on n trials with success probability p if and only if

$$p(y) = \binom{n}{y} p^y q^{n-y}, \qquad y = 0, \dots, n, \qquad 0 \le p \le 1$$

Why?

$$S = \{ \underbrace{SSFFFS \dots FFS}_{n}, \dots \}$$

$$P(\underbrace{SSS \dots S}_{y}\underbrace{FFF \dots F}_{n-y}) = p^y (1-p)^{n-y} = p^y q^{n-y}$$

$$\binom{n}{y} = \# \text{ of } n\text{-tuples} \Rightarrow \boxed{p(y) = \binom{n}{y} p^y q^{n-y}}$$

The term '**binomial**' experiment derives from the fact that each trial results in one of two possible outcomes and that the probabilities $p(y)$ are terms of the binomial expansion:

$$\underbrace{(p+q)}_{1}{}^n = \binom{n}{0} p^0 q^n + \binom{n}{1} p^1 q^{n-1} + \cdots + \binom{n}{n} p^n q^0$$

$$1 \Rightarrow \sum_{y=0}^{n} p(y) = 1$$

Example: A lot of 5000 electrical fuses contains 5% defectives. If a sample of 5 fuses is tested, find P(at least one is defective).

Solution: $P(\text{at least one}) = 1 - P(\text{none})$

$$= 1 - p(0) = 1 - \binom{5}{0} 0.05^0 \cdot 0.95^5$$

$n = 5$

$y = 0$

$$= 0.226$$

Theorem: Let $Y \sim \text{Bin}(n, p)$. Then $\mu = E(Y) = np$ and $\sigma^2 = Var(Y) = npq$.

Proof: $E(Y) = \sum_{y=0}^{n} y\, p(y) = \sum_{y=0}^{n} y \binom{n}{y} p^y q^{n-y}$

$$= \sum_{y=1}^{n} y \frac{n!}{(n-y)!\, y!} p^y q^{n-y} = np \sum_{y=1}^{n} \frac{(n-1)!}{(n-y)!\,(y-1)!} p^{y-1} q^{n-y}$$

$$= (z = y-1) = np \underbrace{\sum_{z=0}^{n-1} \underbrace{\frac{(n-1)!}{(n-1-z)!\, z!} p^z q^{n-1-z}}_{\text{pf for Bin}(n-1,p)}}_{=1}$$

$$\Rightarrow E(Y) = np$$

$$Var(Y) = E(Y^2) - E(Y)^2 = E(Y^2) - (np)^2$$

$$E(Y(Y-1)) = E(Y^2 - Y) = E(Y^2) - E(Y)$$

$$E(Y^2) = E(Y(Y-1)) + E(Y)$$

$$E(Y(Y-1)) = \sum_{y=0}^{n} y(y-1) \binom{n}{y} p^y q^{n-y}$$

$$= \sum_{y=2}^{n} y(y-1) \frac{n!}{(n-y)!\, y!} p^y q^{n-y}$$

$$= p^2 n(n-1) \sum_{y=2}^{n} \frac{(n-2)!}{(n-y)!\,(y-2)!} p^{y-2} q^{n-y}$$

$$= (z = y-2) = p^2 n(n-1) \underbrace{\sum_{z=0}^{n-2} \underbrace{\frac{(n-2)!}{(n-2-z)!\, z!} p^z q^{n-2-z}}_{Bin(n-2,\,p)}}_{=1}$$

$$= p^2 n(n-1)$$

$$E(Y^2) = p^2 n(n-1) + pn$$

$$Var(Y) = E(Y^2) - E(Y)^2$$

$$= p^2 n(n-1) + pn - (pn)^2 = -p^2 n + pn = npq \quad \blacksquare$$

Example: (#3.56) An oil exploration firm is formed with enough capital to finance 10 explorations. The probability of a particular exploration being successful is 0.1. Find μ and σ^2 of the number of successful explorations.

Solution: Y = # of successful exploration

$$Y \sim Bin(\underset{n}{10}, \underset{p}{0.1})$$

$$E(Y) = np = 1$$

$$Var(Y) = npq = 10 \cdot 0.1 \cdot 0.9 = 0.9$$

Geometric Probability Distribution

Let Y be the number of the trial when the 1st success occurs.

S: $E_1 = S$

$E_2 = FS$

$E_3 = FFS$

$\vdots$

$E_k = \underbrace{FF\ldots F}_{k-1} S$

$\vdots$

$$p(y) = P(Y=y) = P(\underbrace{FF\ldots F}_{y-1} S) = q^{y-1} p$$

Definition: A r.v. Y is said to have a **geometric probability distribution** if and only if

$$p(y) = q^{y-1}p, \ \ y = 1,2,3, \dots , \ \ 0 \le p \le 1$$

Let's show that $\sum_y p(y) = 1$.

$$\sum_{y=1}^{\infty} p(y) = \sum_{y=1}^{\infty} q^{y-1} p = p \sum_{y=1}^{\infty} q^{y-1} = p \sum_{y=0}^{\infty} q^{y}$$

$$\sum_{y=0}^{\infty} q^{y} = \left(\text{geom. series } q \le 1\right) = \frac{1}{1-q} = \frac{1}{p} \Rightarrow \sum p(y) = p \cdot \frac{1}{p} = 1 \quad \blacksquare$$

Example: (#3.70) An oil prospector will drill a succession of holes in a given area to find a productive well. The probability of success is 0.2.

(a) What is the probability that the 3rd hole drilled is the first to yield a productive well?

(b) If the prospector can afford to drill at most 10 wells, what is the probability that he will fail to find a productive well?

Solution: (a) $P(Y=3) = 0.8^{2}(0.2) = 0.128$

(b) $P(Y>10) = P(\text{first 10 wells are not productive}) = 0.8^{10}$

Theorem: Let $Y \sim$ Geom(p), then

$$\mu = E(Y) = \frac{1}{p} \text{ and } \sigma^2 = Var(Y) = \frac{1-p}{p^2}$$

Proof:

$$E(Y) = \sum y p(y) = \sum_{y=1}^{\infty} y q^{y-1} p = p \sum_{y=1}^{\infty} y q^{y-1}$$

$$= p \sum_{y=1}^{\infty} \frac{d}{dq}(q^y) = p \frac{d}{dq}\left(\sum_{y=1}^{\infty} q^y\right) =$$

$$-1 + \sum_{y=0}^{\infty} q^y = \frac{1}{1-q} - 1 \Rightarrow \sum_{y=1}^{\infty} q^y = \frac{q}{1-q}$$

$$= \quad p \frac{d}{dq} \frac{q}{1-q} = p \cdot \frac{(1-q)+q}{(1-q)^2} = \frac{p}{(1-q)^2} = \frac{1}{p}$$

$$E(Y(Y-1)) = \sum_{y=1}^{\infty} y(y-1) q^{y-1} p = pq \sum_{y=1}^{\infty} y(y-1) q^{y-2}$$

$$= pq \sum \frac{d^2}{dq^2}(q^y) = pq \frac{d^2}{dq^2} \sum_{y=2}^{\infty} q^y =$$

$$-1-q+\sum_{y=0}^{\infty} q^y = \frac{1}{1-q} - 1 - q \Rightarrow \sum_{y=2}^{\infty} q^y = \frac{1}{1-q} - 1 - q$$

$$= \quad pq \frac{d}{dq}\left(\frac{1}{(1-q)^2} - 1\right) = pq \frac{2}{(1-q)^3} = \frac{2q}{p^2}$$

$$Var(Y) = E(Y^2) - E(Y)^2 = E(Y(Y-1)) + E(Y) - E(Y)^2$$

$$= \frac{2q}{p^2} + \frac{1}{p} - \frac{1}{p^2} = \frac{2q+p-1}{p^2} = \frac{q}{p^2} = \frac{1-p}{p^2}$$

Hypergeometric Probability Distribution

Suppose we have a population of N elements that possess one of two characteristics, e.g. r of them are red and N-r are green.

A sample of n elements is randomly selected from the population. The r.v. of interest, Y, is the number of red elements in the sample.

Definition: A r.v. Y is said to have a **hypergeometric probability distribution** if and only if

$$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}},$$

$$y = 0, \dots, n, \qquad y \le n, \qquad n - y \le N - r$$

Theorem: If Y is a r.v. with a hypergeometric distribution, then

$\mu = E(Y) = \frac{nr}{N}$ and $\sigma^2 = Var(Y) = n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$

Proof: omitted

Example: (#3.102) An urn contains 10 marbles: 2 blue, 5 green, 3 red. Three marbles are to be drawn without replacement. Find P(all 3 marbles are green).

Solution: $P(Y=3) = \dfrac{\binom{5}{3}\binom{5}{0}}{\binom{10}{3}}$

$= \dfrac{\frac{5!}{3!\,2!}}{\frac{10!}{3!\,7!}} = ?$

Poisson Probability Distribution

Definition: A r.v. Y is said to have a **Poisson probability distribution** with parameter λ if and only if

$$p(y) = \frac{\lambda^y e^{-\lambda}}{y!}, \qquad y = 0,1,2,\dots, \qquad \lambda > 0$$

We can show that it is the limit of the binomial distribution, i.e.

$$\lim_{n\to\infty}\binom{n}{y}p^y(1-p)^{n-y} = \frac{\lambda^y e^{-\lambda}}{y!}, \quad \lambda = np \Rightarrow p = \frac{\lambda}{n}$$

$$\lim_{n\to\infty}\frac{n!}{(n-y)!\,y!}\left(\frac{\lambda}{n}\right)^y\left(1-\frac{\lambda}{n}\right)^{n-y} = \lim_{n\to\infty}\frac{n(n-1)(n-2)\cdots(n-y+1)}{y!}\left(\frac{\lambda}{n}\right)^y\left(1-\frac{\lambda}{n}\right)^{n-y}$$

$$= \lim_{n\to\infty}\frac{\lambda^y}{y!}\underbrace{\left(1-\frac{\lambda}{n}\right)^{n-y}}\,\frac{n(n-1)(n-2)\cdots(n-y+1)}{n^y}$$

$$= \frac{\lambda^y}{y!}\lim\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-y}\frac{\cancel{n}(n-1)(n-2)\cdots(n-y+1)}{\cancel{n}\cdot n\cdot n\cdots n}$$

$$= \frac{\lambda^y}{y!}\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n\underbrace{\left(1-\frac{\lambda}{n}\right)^{-y}}_{\to 1}\underbrace{\left(1-\frac{1}{n}\right)}_{\to 1}\underbrace{\left(1-\frac{2}{n}\right)}_{\to 1}\cdots\underbrace{\left(1-\frac{y-1}{n}\right)}_{\to 1}$$

$$= \frac{\lambda^y}{y!}\boxed{\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n} = \frac{\lambda^y}{y!}e^{-\lambda} \quad \blacksquare$$

$$\sum_{y=0}^{\infty}p(y) = \sum_{y=0}^{\infty}\frac{\lambda^y e^{-\lambda}}{y!} = e^{-\lambda}\underbrace{\sum_{y=0}^{\infty}\frac{\lambda^y}{y!}}_{e^{\lambda}} = 1$$

$$\boxed{\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x}$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = e^x$$

<u>Theorem</u>: If $Y \sim$ Poisson(λ), then

$\mu = E(Y) = \lambda$ and $\sigma^2 = Var(Y) = \lambda$.

<u>Proof</u>:

$$E(Y) = \sum y\, p(y) = \sum_{y=0}^{\infty} y \frac{\lambda^y e^{-\lambda}}{y!}$$

$$= \sum_{y=1}^{\infty} \frac{\lambda^y e^{-\lambda}}{(y-1)!} = \lambda \sum_{y=1}^{\infty} \frac{\lambda^{y-1} e^{-\lambda}}{(y-1)!}$$

$$= (z = y-1) = \lambda \underbrace{\sum_{z=0}^{\infty} \frac{\lambda^z e^{-\lambda}}{z!}}_{=1} = \lambda$$

$$Var(Y) = E(Y^2) - E(Y)^2 = E(Y(Y-1)) + E(Y) - E(Y)^2$$

$$E(Y(Y-1)) = \sum_{y=0}^{\infty} y(y-1) \frac{\lambda^y e^{-\lambda}}{y!} = \sum_{y=2}^{\infty} \frac{\lambda^y e^{-\lambda}}{(y-2)!}$$

$$= \lambda^2 \sum_{y=2}^{\infty} \frac{\lambda^{y-2} e^{-\lambda}}{(y-2)!} = (z = y-2)$$

$$= \lambda^2 \underbrace{\sum_{z=0}^{\infty} \frac{\lambda^z e^{-\lambda}}{z!}}_{=1} = \lambda^2$$

$$Var(Y) = \lambda^2 + \lambda - \lambda^2 = \lambda$$

$\blacksquare$

Example: (#3.122) Customers arrive at a checkout counter according to a Poisson distribution at an average of 7 per hour. During a given hour, what are the probabilities that

(a) No more than 3 customers arrive?
(b) At least 2 customers arrive?
(c) Exactly 5 customers arrive?

Solution: $Y \sim \text{Poisson}(7)$

(a) $P(Y \le 3) = p(0) + p(1) + p(2) + p(3)$

$$= \frac{7^0 e^{-7}}{0!} + \frac{7^1 e^{-7}}{1!} + \frac{7^2 e^{-7}}{2!} + \frac{7^3 e^{-7}}{3!}$$

$$= 0.0818$$

(b) $P(Y \geq 2) = 1 - P(Y \le 1)$

$$= 1 - p(0) - p(1) = 0.9927$$

(c) $P(Y = 5) = p(5) = \dfrac{7^5 e^{-7}}{5!}$

$$= 0.1277$$

Moments and Moment-Generating Functions

Definition: The $\boldsymbol{k^{th}}$ **moment** of a r.v. Y taken about its mean, or the $\boldsymbol{k^{th}}$ **central moment** of Y, is defined to be $E[(Y-\mu)^k]$ and denoted by μ_k.

$$\boxed{\mu_k' = E(Y^k)}$$

$E(Y) = \mu$

$E(Y^2)$

$\vdots$

Note: $\sigma^2 = \mu_2$.

Definition: The **moment-generating function** (mgf), $m(t)$, for a r.v. Y is defined to be $m(t) = E(e^{tY})$.

We say that an mgf for Y exists if there is $b > 0$ such that

$m(t) < \infty$ for $|t| \le b$.

$$e^X = 1 + X + \frac{X^2}{2!} + \frac{X^3}{3!} + \cdots$$

$$e^{tY} = 1 + tY + \frac{(tY)^2}{2!} + \frac{(tY)^3}{3!} + \cdots$$

$$m(t) = E(e^{tY}) = \sum_y e^{ty} p(y)$$

$$= \sum_y \left[1 + ty + \frac{(ty)^2}{2!} + \frac{(ty)^3}{3!} + \cdots\right] p(y)$$

$$= \sum \left[p(y) + ty\,p(y) + \frac{(ty)^2}{2!} p(y) + \cdots\right]$$

$$= \sum p(y) + t \sum y\,p(y) + \frac{t^2}{2!} \sum y^2 p(y) + \cdots$$

$$= \boxed{1 + tE(Y) + \frac{t^2}{2!} E(Y^2) + \cdots}$$

$\longrightarrow$ function of all moments

Theorem: If $m(t)$ exists, then for any $k \in Z^+$

$$\frac{d^k m(t)}{dt^k}\Big|_{t=0} = m^{(k)}(0) = \mu'_k = E(Y^k)$$

Proof:

$$m(t) = 1 + t\mu_1' + \frac{t^2}{2!}\mu_2' + \frac{t^3}{3!}\mu_3' + \dots$$

$$m'(t) = \mu_1' + \frac{2t}{2!}\mu_2' + \frac{3t^2}{3!}\mu_3' + \dots$$

$$m'(0) = \mu_1' = E(Y)$$

$$m''(t) = \mu_2' + \frac{2t}{2!}\mu_3' + \dots$$

$$m''(0) = \mu_2' = E(Y^2)$$

Example: Find the mgf for $Y \sim \text{Poisson}(\lambda)$.

Solution:

$$m(t) = E(e^{tY}) = \sum_{y=0}^{\infty} e^{ty}\frac{\lambda^y e^{-\lambda}}{y!}$$

$$= e^{-\lambda}\sum_{y=0}^{\infty}\frac{(e^t\lambda)^y}{y!} = e^{-\lambda}\cdot e^{e^t\lambda}$$

$$= e^{\lambda(e^t-1)}$$

$$E(Y) = m'(0) = e^{\lambda(e^t-1)}\cdot\lambda e^t\Big|_{t=0} = \lambda$$

Example: (#3.155) Let $m(t) = \frac{1}{6}e^{t} + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}$. Find

(a) $E(Y)$;
(b) Var(Y);
(c) Distribution of Y.

Solution:

(a) $$E(Y) = m'(0) = \frac{1}{6}e^{t} + \frac{2}{6}e^{2t}\cdot 2 + \frac{3}{6}e^{3t}\cdot 3\Big|_{t=0}$$

$$= \frac{1}{6} + \frac{4}{6} + \frac{9}{6} = \frac{7}{3}$$

(b) $$\mathrm{Var}(Y) = E(Y^2) - E(Y)^2 = 6 - \frac{49}{9} = \frac{5}{9}$$

$$E(Y^2) = m''(0) = \frac{1}{6}e^{t} + \frac{2}{6}e^{2t}\cdot 4 + \frac{3}{6}e^{3t}\cdot 9\Big|_{t=0}$$

$$= \frac{1}{6} + \frac{8}{6} + \frac{27}{6} = 6$$

(c) $$m(t) = E(e^{tY}) = \sum e^{ty}p(y)$$

$$= e^{ty_1}p(y_1) + e^{ty_2}p(y_2) + \dots$$

Y:	1	2	3
	1/6	2/6	3/6

$$E(Y) = 1\cdot\frac{1}{6} + 2\cdot\frac{2}{6} + 3\cdot\frac{3}{6} = \frac{7}{3}$$