F-Tests: Overview

https://en.wikipedia.org/wiki/Fuddle_duddle

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Review: χ^2 and F

- If $Z_1, Z_2, \ldots, Z_N \sim N(0,1)$ are iid, then

$$\sum Z_i^2 \sim \chi^2(N)$$

- If $W_1 \sim \chi^2(k_1)$ and $W_2 \sim \chi^2(k_2)$, then

$$F = \frac{W_1/k_1}{W_2/k_2} \sim F(k_1, k2)$$

Are All Non-Intercept Coefficients 0?

- $\hat{Y}_i = \beta_0 + \beta_1 X_i^{(1)} + \beta_2 X_i^{(2)} + \cdots + \beta_{p-1} X_i^{(p-1)}$
- $SST = \sum_{i=1}^{n} (Y_i - \bar{Y})^2$ (*n-1* df)
- $SSE = \sum_{i=1}^{n} \left(Y_i - \hat{Y}_i\right)^2$ (*n-p* df)
  - "Within-group variation" when the X's indicate membership in groups
- $SSR = SST - SSE = \sum_{i=1}^{n} \left(\hat{Y}_i - \bar{Y}\right)^2$ (*p-1* df)
  - "Between-group variation" when the X's indicate membership in groups

Are All Non-Intercept Coefficients 0?

- *If* all non-intercept coefficients are 0 (i.e., all group means are the same if the X's indicate membership in groups):
  - $1/\sigma^2 SST = 1/\sigma^2 \sum_{i=1}^{n}(Y_i - \bar{Y})^2 \sim \chi^2(n-1)$
  - $1/\sigma^2 SSE = 1/\sigma^2 \sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2 \sim \chi^2(n-p)$
  - $1/\sigma^2 SSR = 1/\sigma^2 \sum_{i=1}^{n}\left(\hat{Y}_i - \bar{Y}\right)^2 \sim \chi^2(p-1)$
- $F = \frac{SSR/(p-1)}{SSE/(n-p)} \sim F(p-1, n-p)$
- **Reject** the hypothesis that **all** means are the same/**all** non-intercept coefs. Are 0 if the p-value is small
- Cannot reject the hypothesis that any particular one of the coefs. is non-zero (unless there is just the one coefficient)
- P-value large: cannot make any particular conclusion

Partial F-tests

- Null Hypothesis: all the coefficients in the Full model that are not in the Reduced model are 0
- Then:
- $\frac{1}{\sigma^2} SSE_{Full} = \frac{1}{\sigma^2} \sum_i \left(Y_i - \hat{Y}_i^{Full}\right)^2 \sim \chi^2(n-p)$
- $\frac{1}{\sigma^2} SSE_{Reduced} = \frac{1}{\sigma^2} \sum_i \left(Y_i - \hat{Y}_i^{Reduced}\right)^2 \sim \chi^2(n-p-p_1)$
- $\frac{1}{\sigma^2} (SSE_{Reduced} - SSE_{Full}) \sim \chi^2(p_1)$
- $F = \frac{(SSE_{Reduced} - SSE_{Full})/p_1}{SSE_{Full}/(n-p)} \sim F(p_1, n-p)$

(In R, two ways of doing F-tests)

Slide from Monday

```
> anova(fit_saturated)
Analysis of Variance Table

Response: Score
              Df Sum Sq Mean Sq F value  Pr(>F)
Treat          1 327.34  327.34  6.3080 0.03323 *
Company        9 682.52   75.84  1.4614 0.29051
Treat:Company  9 311.46   34.61  0.6669 0.72212
Residuals      9 467.04   51.89
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

- Can we say that `Treat` is significant?
  - Only if we planned the study that way all along
    - Usually we plan to do a t-test on the final model, like we did on Monday
  - Planning a study like that is weird… What’s the hypothesis there, and why the interaction there?
  - But there is evidence there that `Treat` is important