

STA 261s2005 Assignment 9

Do this assignment in preparation for the quiz on Wednesday, March 23d. The questions are practice for the quiz, and are not to be handed in.

Please start by reading Section 12.4 in the text (skip 12.3), and also read the last part of the online supplement — the material on uniformly most powerful tests and the Neyman-Pearson Lemma.

1. Let $Y_i = x_i + \epsilon_i$, for $i = 1, \dots, n$, where
 - x_1, \dots, x_n are fixed, known constants
 - $\epsilon_1, \dots, \epsilon_n$ are independent and identically distributed $\text{Normal}(0, \sigma^2)$ random variables; the parameter σ^2 is unknown.
 - The data consist of n pairs (x_i, Y_i) . The error terms ϵ_i are not given directly.
 - (a) Find the distribution of $\frac{1}{\sigma^2} \sum_{i=1}^n (Y_i - x_i)^2$.
 - (b) We wish to test $H_0 : \sigma^2 \leq \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$.
 - i. What is Θ ?
 - ii. What is \mathcal{X} ?
 - iii. What is Θ_0 ? Is it simple or composite?
 - iv. What is Θ_1 ? Is it simple or composite?
 - (c) Find the constant k so that the following test will be size α for testing the *simple* null hypothesis $H_0 : \sigma^2 = \sigma_0^2$.
$$C = \{\mathbf{x} \in \mathcal{X} : \frac{\sum_{i=1}^n (Y_i - x_i)^2}{\sigma_0^2} > k\}$$
 - (d) Find the power function $P_{\sigma^2}\{\mathbf{X} \in C\} = \pi(\sigma^2)$.
 - (e) Prove that the test C is also size α for testing $H_0 : \sigma^2 \leq \sigma_0^2$.
2. Do exercises 12.1, 12.5, 12.6 (Answ: 0.223 and 0.451), 12.7, 12.8 (Answ: 0 and $\frac{\beta_0+1}{\beta_0+2}$).
3. Do exercise 12.9, as follows:
 - (a) Sketch the sample space \mathcal{X} .
 - (b) Put the critical region C in the picture. It is a curvy region in the upper right-hand corner of \mathcal{X} .
 - (c) Obtain the answer by directly integrating over C .
4. Do exercises 12.10 through 12.15.