Name ______________________________

Student Number ______________________________

STA 260 S 2020 Test 1A

Tutorial Section (Circle One)

TUT0101 Tues. 3-4 IB 200 Dashvin	TUT0102 Tues. 4-5 IB 200 Karan	TUT0103 Wed. 5-6 IB 220 Marie	TUT0104 Wed. 5-6 IB 200 Karan
TUT0105 Wed. 6-7 DV 1148 Karan	TUT0106 Fri. 3-4 IB 200 Michael	TUT0107 Fri. 4-5 IB 200 Michael	TUT0108 Fri. 5-6 IB 200 Marie

Question	Value	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total = 100 Points		

1. (20 points) The continuous random variable X has density $f_X(x) = -\ln(x)\, I(0 < x < 1)$. Let $Y = -\ln(X)$. Find the density $f_Y(y)$. Show your work and **circle your final answer**.

Continued on page 3

2. (20 points) Let $X_1, \ldots, X_n$ be independent discrete random variables with probability mass function $p_X(x|\theta) = \theta\, I(x = 3) + (1 - \theta)\, I(x = 5)$. A proposed estimator is $\widehat{\Theta}_n = \frac{1}{2}(5 - \overline{X}_n)$.

   (a) Is $\widehat{\Theta}_n$ unbiased? Write **Biased**, **Unbiased** or **Impossible to answer**, and show your work. If it's impossible to answer, explain why.

   (b) Is $\widehat{\Theta}_n$ consistent? Write **Consistent**, **Not consistent** or **Impossible to answer**, and show your work. If it's impossible to answer, explain why.

Continued on page 4

3. Let $X_1, \dots, X_n$ be independent but *not* identically distributed random variables, with $X_i \sim \text{Gamma}(\alpha, \lambda_i)$. The parameter α is unknown, while $\lambda_1, \dots, \lambda_n$ are known constants. A proposed estimator is $\widehat{\alpha}_n = \frac{1}{n} \sum_{i=1}^{n} \lambda_i X_i$.

   (a) (8 points) Is $\widehat{\alpha}_n$ unbiased? Write **Biased**, **Unbiased** or **Impossible to answer**, and show your work. If it's impossible to answer, explain why.

   (b) (12 points) Is $\widehat{\alpha}_n$ consistent? Write **Consistent**, **Not consistent** or **Impossible to answer**, and show your work. If it's impossible to answer, explain why.

Continued on page 5

4. Let $X_1, \ldots, X_n$ be a random sample from a Normal(μ, σ^2) distribution.

   (a) (15 points) Find the distribution of the sample mean $\overline{X}_n$. Show your work. **Circle the final answer, which includes the parameters of the distribution**.

Continued on page 6

Continue your answer to Question 4a if necessary.

(b) (5 points) State the distribution of $W = \frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}}$, including the parameters. Justify your answer by quoting a single fact from the formula sheet.

Continued on page 7

5. Let $X_1, \ldots, X_n$ be a random sample from a Normal($\mu, \sigma^2 = 4$) distribution. The expected value μ is unknown, but the variance is known to be $\sigma^2 = 4$.

   (a) (15 points) Derive an *exact* $(1-\alpha)100\%$ confidence interval for μ. Exact means the probability that the interval will contain μ is *exactly* $1-\alpha$, and you are *not* using the Central Limit Theorem. You are also not using the t distribution, because that's after Test One. Your final answer is two formulas, one for the upper confidence limit, and one for the lower confidence limit. Show your work. **Circle the formulas.**

Continued on page 8

Continue Question 5a if necessary.

(b) (5 points) Still for the normal model with unknown μ and $\sigma^2 = 4$, a random sample of size $n = 125$ yields a sample mean of $\overline{x}_n = 18.2$. Give a 95% confidence interval for μ. Your answer is a pair of numbers, a lower confidence limit and an upper confidence limit. **Circle your answer.**

Total Marks = 100 points