Sample Questions: Transformations

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1. Let $X \sim$ Poisson(λ_1) and $Y \sim$ Poisson(λ_2) be independent. Using the convolution formula, find the probability mass function of $Z = X + Y$ and identify it by name.

2. Independently for $i = 1, \ldots, n$, let $X_i \sim \text{Poisson}(\lambda_i)$, and let $Y_n = \sum_{i=1}^{n} X_i$. Using the last problem, what is the probability distribution of Y_n?

3. Let $X \sim \text{Binomial}(n_1, \theta)$ and $Y \sim \text{Binomial}(n_2, \theta)$ be independent. Using the convolution formula, find the probability mass function of $Z = X + Y$ and identify it by name.

4. Let $X_1, \ldots, X_n$ be independent Bernoulli random variables with parameter θ, and let $Y_n = \displaystyle\sum_{i=1}^{n} X_i$. Using the last problem, what is the probability distribution of Y?

5. Let X and Y be independent exponential random variables with parameter λ. Using the convolution formula, find the probability density function of $Z = X + Y$ and identify it by name.

6. Let X_1 and X_2 be independent standard normal random variables. Find the probability density function of $Y_1 = X_1/X_2$.

7. Use the Jacobian method to prove the convolution formula for continuous random variables.

8. Prove $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

9. Show that the normal probability density function integrates to one.
