

## Sample Questions: Transformations

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1. Let  $X \sim \text{Poisson}(\lambda_1)$  and  $Y \sim \text{Poisson}(\lambda_2)$  be independent. Using the convolution formula, find the probability mass function of  $Z = X + Y$  and identify it by name.

2. Independently for  $i = 1, \dots, n$ , let  $X_i \sim \text{Poisson}(\lambda_i)$ , and let  $Y_n = \sum_{i=1}^n X_i$ . Using the last problem, what is the probability distribution of  $Y_n$ ?

3. Let  $X \sim \text{Binomial}(n_1, \theta)$  and  $Y \sim \text{Binomial}(n_2, \theta)$  be independent. Using the convolution formula, find the probability mass function of  $Z = X + Y$  and identify it by name.

4. Let  $X_1, \dots, X_n$  be independent Bernoulli random variables with parameter  $\theta$ , and let  $Y_n = \sum_{i=1}^n X_i$ . Using the last problem, what is the probability distribution of  $Y$ ?

5. Let  $X$  and  $Y$  be independent exponential random variables with parameter  $\lambda$ . Using the convolution formula, find the probability density function of  $Z = X + Y$  and identify it by name.

6. Let  $X_1$  and  $X_2$  be independent standard normal random variables. Find the probability density function of  $Y_1 = X_1/X_2$ .

7. Use the Jacobian method to prove the convolution formula for continuous random variables.

8. Prove  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ .



9. Show that the normal probability density function integrates to one.

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<http://www.utstat.toronto.edu/~brunner/oldclass/256f19>