# Sample Questions: Foundations of Probability

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1. Prove Property 5: $P(A^c) = 1 - P(A)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

2. Prove Property 6: If $A \subseteq B$ then $P(A) \leq P(B)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

3. Prove Property 7 (the inclusion-exclusion principle): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

4. If 23 out of 25 are employed, what is the probability of randomly choosing an unemployed person? The answer is a number. Circle your answer.

5. If you roll two fair dice, what is the probability of getting a sum greater than 2? The answer is a number. Circle your answer.

6. If you roll two fair dice, what is the probability of getting two different numbers? Your answer is a number. Circle your answer.

7. $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$. What is $P(A \cup B)$? The answer is a number. Circle your answer.

8. Of the cars in a used car lot, 50% have engine trouble and 50% have transmission trouble. If 25% have both problems and you buy a car at random, what is the probability that both the engine and transmission are okay? The answer is a number. Circle your answer.

9. Let $A_1, A_2, \ldots$ form a partition of the sample space $S$, meaning that $A_1, A_2, \ldots$ are disjoint and $S = \cup_{k=1}^{\infty} A_k$. Let $B$ be any event. Show that $P(B) = \sum_{k=1}^{\infty}(A_k \cap B)$. Use the 4 basic properties of probability and the tabular format illustrated in lecture.

10. Let $A_1, A_2, \ldots$ be a collection of events, not necessarily disjoint. Show that $P\left(\cup_{k=1}^{\infty} A_k\right) \leq \sum_{k=1}^{\infty} P(A_k)$. Use the Properties 1-7 of probability and the tabular format illustrated in lecture.

11. Let $A_1 \subseteq A_2 \subseteq A_3 \subseteq \ldots$ and let $A = \cup_{k=1}^{\infty} A_k$. Show that $\lim_{k\to\infty} P(A_k) = P(A)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

12. Let $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots$ and let $A = \cap_{k=1}^{\infty} A_k$. Show that $\lim_{k \to \infty} P(A_k) = P(A)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.