Sample Questions: Foundations of Probability

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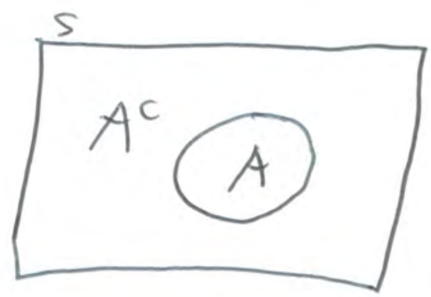
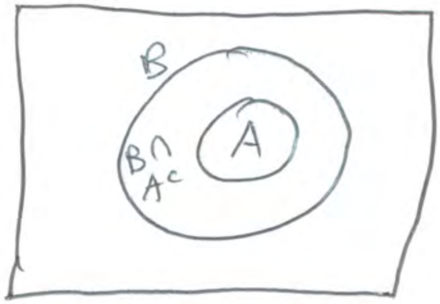
1. Prove Property 5: $P(A^c) = 1 - P(A)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

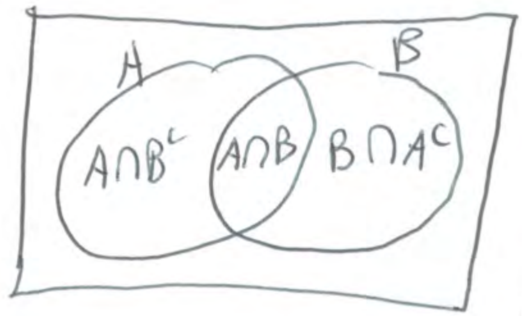
Step	Justification
$S = A \cup A^c$ Disjoint	
$P(S) = P(A) + P(A^c)$	Property 4
$= 1$	Property 3
$\Rightarrow P(A^c) = 1 - P(A)$	Math

$\square$

2. Prove Property 6: If $A \subseteq B$ then $P(A) \leq P(B)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

$B = A \cup (B \cap A^c)$ Disjoint	
$P(B) = P(A) + P(B \cap A^c)$	Property 4
$\Rightarrow P(B) - P(A) = P(B \cap A_c)$	Math
≥ 0	Property 1
$\Rightarrow P(B) \geq P(A)$ $\blacksquare$	Math

3. Prove Property 7 (the inclusion-exclusion principle): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

1	$A \cup B = (A \cap B^c) \cup (A \cap B) \cup (B \cap A^c)$ Disjoint	
2	$A = (A \cap B^c) \cup (A \cap B)$ Disjoint	"
3	$B = (A \cap B) \cup (B \cap A^c)$ Disjoint	"
4	$P(A) = P(A \cap B^c) + P(A \cap B)$	Property 4
5	$P(B) = P(A \cap B) + P(B \cap A^c)$	"
6	$P(A \cup B) = P(A \cap B^c) + P(A \cap B) + P(B \cap A^c)$	"
7	$= P(A \cap B^c) + P(A \cap B) + P(A \cap B) + P(B \cap A^c) - P(A \cap B)$	Math
8	$= P(A) + P(B) - P(A \cap B)$	Steps 4 & 5

4. If 23 out of 25 are employed, what is the probability of randomly choosing an unemployed person? The answer is a number. Circle your answer.

$$1 - P(A) = 1 - \frac{23}{25} = \frac{2}{25}$$

5. If you roll two fair dice, what is the probability of getting a sum greater than 2? The answer is a number. Circle your answer.

	1	2	3	4	5	6
1	2	3				
2	3	4				
3						
4						
5						
6						

$$1 - P(\{2\}) = 1 - \frac{1}{36} = \frac{35}{36}$$

6. If you roll two fair dice, what is the probability of getting two different numbers? Your answer is a number. Circle your answer.

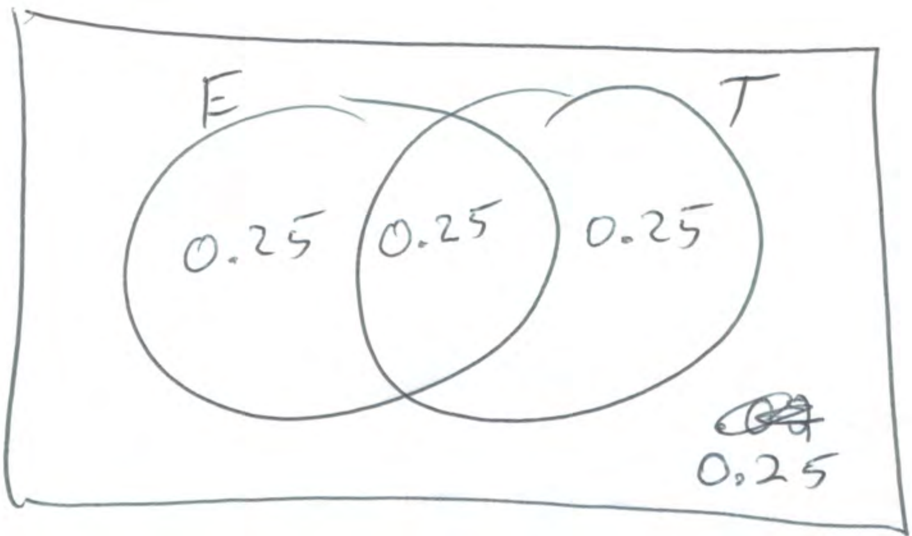
	1	2	3	4	5	6
1	–					
2		–				
3			–			
4				–		
5					–	
6						–

$$\frac{5}{6}$$

7. $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$. What is $P(A \cup B)$? The answer is a number. Circle your answer.

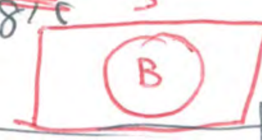
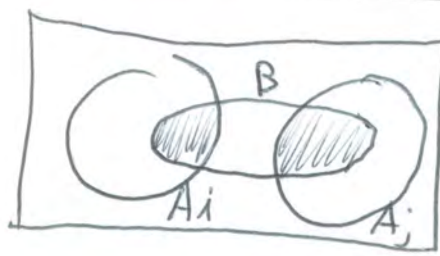
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.4 + 0.5 - 0.3$$
$$= 0.6$$

8. Of the cars in a used car lot, 50% have engine trouble and 50% have transmission trouble. If 25% have both problems and you buy a car at random, what is the probability that both the engine and transmission are okay? The answer is a number. Circle your answer.



0.25

9. Let $A_1, A_2, \ldots$ form a partition of the sample space S, meaning that $A_1, A_2, \ldots$ are disjoint and $S = \cup_{k=1}^{\infty} A_k$. Let B be any event. Show that $P(B) = \sum_{k=1}^{\infty}(A_k \cap B)$. Use the 4 basic properties of probability and the tabular format illustrated in lecture.

$B = B \cap S$	~~Set logic~~ 
$= B \cap \bigcup_{k=1}^{\infty} A_k$	Substitution
$= \bigcup_{k=1}^{\infty} (A_k \cap B)$,	Distributive Law
Disjoint	
$P(B) = \sum_{k=1}^{\infty} P(A_k \cap B)$ $\square$	Property 4

10. Let $A_1, A_2, \ldots$ be a collection of events, not necessarily disjoint. Show that $P\left(\cup_{k=1}^{\infty} A_k\right) \leq \sum_{k=1}^{\infty} P(A_k)$. Use the Properties 1-7 of probability and the tabular format illustrated in lecture.

Let $B_1 = A_1$, $B_2 = A_2 \cap A_1^c$, $B_3 = A_3 \cap (A_1 \cup A_2)^c$

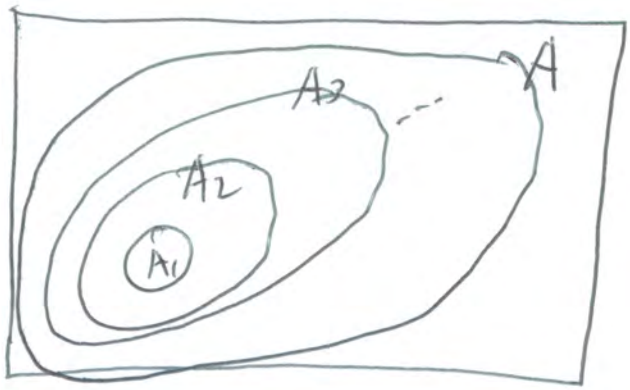
So that each B_k is the part of A_k not in $A_1, \ldots A_{k-1}$

1	$B_1, B_2, \ldots$ Are disjoint, and $\bigcup_{k=1}^{\infty} A_k = \bigcup_{k=1}^{\infty} B_k$	By construction
2	$P\left(\bigcup_{k=1}^{\infty} A_k\right) = P\left(\bigcup_{k=1}^{\infty} B_k\right) = \sum_{k=1}^{\infty} P(B_k)$	(By step 1) & Property 4
3	$B_k \subseteq A_k$, so	By construction
4	$P(B_k) \leq P(A_k)$	Property 6
5	$\Rightarrow \sum_{k=1}^{\infty} P(B_k) \leq \sum_{k=1}^{\infty} P(A_k)$	Math
6	$\Rightarrow P\left(\bigcup_{k=1}^{\infty} A_k\right) \leq \sum_{k=1}^{\infty} P(A_k)$	Step 2

$\boxtimes$

Section 1.6

11. Let $A_1 \subseteq A_2 \subseteq A_3 \subseteq \ldots$ and let $A = \cup_{k=1}^{\infty} A_k$. Show that $\lim_{k\to\infty} P(A_k) = P(A)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.



Set $B_1 = A_1$
$B_2 = A_2 \cap A_1^c$
$B_3 = A_3 \cap A_2^c$
$\vdots$

Rings

1	$A_n = \bigcup_{k=1}^{n} B_k$ Disjoint	Picture
2	$P(A_n) = \sum_{k=1}^{n} P(B_k)$	Property 4
	$A = \bigcup_{k=1}^{\infty} B_k$, disjoint	Picture
	$P(A) = \sum_{k=1}^{\infty} P(B_k)$	Property 4
	$= \lim_{n\to\infty} \sum_{k=1}^{n} P(B_k)$	Def of ∞ sum
	$= \lim_{n\to\infty} P(A_n)$ $\blacksquare$	Step 2

12. Let $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots$ and let $A = \cap_{k=1}^{\infty} A_k$. Show that $\lim_{k \rightarrow \infty} P(A_k) = P(A)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

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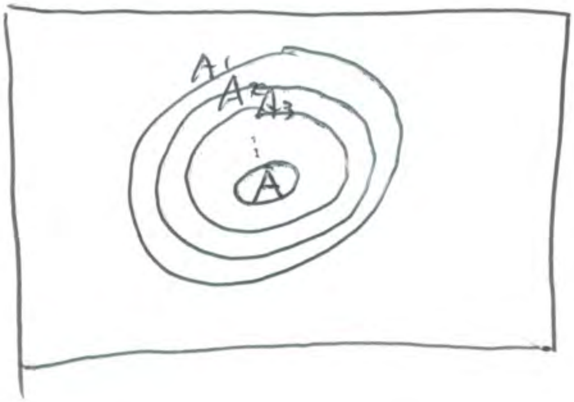
http://www.utstat.toronto.edu/~brunner/oldclass/256f19



Let $B_1 = A_1^c$
$B_2 = A_2^c \cap A_1$
$B_3 = A_3^c \cap A_2$
etc.

Rings

1	$A^c = \bigcup_{k=1}^{\infty} B_k$, Disjoint	Picture
2	$A_n^c = \bigcup_{k=1}^{n} B_k$, disjoint	"
3	$P(A_n^c) = \sum_{k=1}^{n} P(B_k)$	Property 4
4	$P(A^c) = \sum_{k=1}^{\infty} P(B_k)$	"
5	$= \lim_{n\to\infty} \sum_{k=1}^{n} P(B_k)$	Def of ∞ sum
6	$= \lim_{n\to\infty} P(A_n^c)$	Step 3
	$= \lim_{n\to\infty} (1 - P(A_n))$	Property 5
	$= 1 - \lim_{n\to\infty} P(A_n)$	Math
	$\Rightarrow 1 - P(A) = 1 - \lim_{n\to\infty} P(A_n)$	Step 6
	$\Rightarrow \lim_{n\to\infty} P(A_n) = P(A)$	Math

$\square$