

# Sample Questions: Conditional Distributions and Independent Random Variables

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1. Let  $X$  and  $Y$  be continuous random variables. Prove that  $X$  and  $Y$  are independent if and only if  $f_{X,Y}(x, y) = f_X(x) f_Y(y)$ .

2. Let  $X$  and  $Y$  be discrete random variables. Prove that if  $p_{X,Y}(x,y) = p_X(x)p_Y(y)$ , then  $X$  and  $Y$  are independent.

3. Let  $X$  and  $Y$  be discrete random variables. Prove that if  $X$  and  $Y$  are independent, then  $p_{X,Y}(x,y) = p_X(x)p_Y(y)$ .

4. Let  $p_{X,Y}(x, y) = \frac{|x-2y|}{20}$  for  $x = 1, 2, 3$  and  $y = 1, 2, 3$ , and zero otherwise.

(a) What is  $p_{Y|X}(1|2)$ ?

(b) What is  $p_{X|Y}(1|2)$ ?

(c) Are  $X$  and  $Y$  independent? Answer Yes or No and prove your answer.

5. Let  $f_{X,Y}(x, y) = \begin{cases} 2e^{-(x+y)} & \text{for } 0 \leq x \leq y \text{ and } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$

(a) Find  $f_{X|Y}(x|y)$ .

(b) Are  $X$  and  $Y$  independent? Answer Yes or No and prove your answer.

6. Let  $X_1, \dots, X_n$  be independent random variables with probability density function  $f_X(x)$  and cumulative distribution function  $F_X(x)$ . Let  $Y = \max(X_1, \dots, X_n)$ . Find the density  $f_Y(y)$ .

7. Let  $X_1, \dots, X_n$  be independent random variables with probability density function  $f_X(x) = e^{-x}$  for  $x \geq 0$ . Let  $Y = \max(X_1, \dots, X_n)$ . Find the density  $f_Y(y)$ .

8. Let  $X_1, \dots, X_n$  be independent random variables with probability density function  $f_X(x)$  and cumulative distribution function  $F_X(x)$ . Let  $Y = \min(X_1, \dots, X_n)$ . Find the density  $f_Y(y)$ .

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<http://www.utstat.toronto.edu/~brunner/oldclass/256f19>