Sample Questions: Conditional Distributions and Independent Random Variables

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1. Let X and Y be continuous random variables. Prove that X and Y are independent if and only if $f_{X,Y}(x,y) = f_X(x)\, f_Y(y)$.

2. Let X and Y be discrete random variables. Prove that if $p_{X,Y}(x, y) = p_X(x)\, p_Y(y)$, then X and Y are independent.

3. Let X and Y be discrete random variables. Prove that if X and Y are independent, then $p_{X,Y}(x, y) = p_X(x)\, p_Y(y)$.

4. Let $p_{X,Y}(x,y) = \frac{|x-2y|}{20}$ for $x = 1,2,3$ and $y = 1,2,3$, and zero otherwise.

(a) What is $p_{Y|X}(1|2)$?

(b) What is $p_{X|Y}(1|2)$?

(c) Are X and Y independent? Answer Yes or No and prove your answer.

5. Let $f_{X,Y}(x, y) = \begin{cases} 2e^{-(x+y)} & \text{for } 0 \leq x \leq y \text{ and } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$

(a) Find $f_{X|Y}(x|y)$.

(b) Are X and Y independent? Answer Yes or No and prove your answer.

6. Let $X_1, \ldots, X_n$ be independent random variables with probability density function $f_X(x)$ and cumulative distribution function $F_X(x)$. Let $Y = \max(X_1, \ldots, X_n)$. Find the density $f_Y(y)$.

7. Let $X_1, \ldots, X_n$ be independent random variables with probability density function $f_X(x) = e^{-x}$ for $x \geq 0$. Let $Y = \max(X_1, \ldots, X_n)$. Find the density $f_Y(y)$.

8. Let $X_1, \ldots, X_n$ be independent random variables with probability density function $f_{_X}(x)$ and cumulative distribution function $F_{_X}(x)$. Let $Y = \min(X_1, \ldots, X_n)$. Find the density $f_{_Y}(y)$.