STA 256 Formulas

$\sum_{k=j}^{\infty} a^k = \frac{a^j}{1-a}$ $\qquad$ $\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$ $\qquad$ $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

$\lim_{x\to c} \frac{g(x)}{h(x)} = \lim_{x\to c} \frac{g'(x)}{h'(x)}$ if $\frac{0}{0}$ or $\frac{\infty}{\infty}$ etc. $\qquad$ $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$

$\Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1}\, dt$ $\qquad$ $\Gamma(\alpha+1) = \alpha\, \Gamma(\alpha)$ $\qquad$ $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

Distributive Laws of Sets: $A \cap \left(\cup_{j=1}^{\infty} B_j\right) = \cup_{j=1}^{\infty}(A \cap B_j)$ $\qquad$ $A \cup \left(\cap_{j=1}^{\infty} B_j\right) = \cap_{j=1}^{\infty}(A \cup B_j)$

De Morgan Laws: $(\cap_{j=1}^{\infty} A_j)^c = \cup_{j=1}^{\infty} A_j^c$ $\qquad$ $(\cup_{j=1}^{\infty} A_j)^c = \cap_{j=1}^{\infty} A_j^c$

Properties of probability:

1. $0 \le P(A) \le 1$ for any $A \subseteq S$
2. $P(\emptyset) = 0$
3. $P(S) = 1$
4. If $A_1, A_2 \ldots$ are disjoint subsets of S, $P\left(\cup_{k=1}^{\infty} A_k\right) = \sum_{k=1}^{\infty} P(A_k)$.
5. $P(A^c) = 1 - P(A)$
6. If $A \subseteq B$ then $P(A) \le P(B)$
7. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
8. If $A_1 \subseteq A_2 \subseteq A_3 \subseteq \ldots$ and $A = \cup_{k=1}^{\infty} A_k$, then $\lim_{k\to\infty} P(A_k) = P(A)$.
9. If $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots$ and $A = \cap_{k=1}^{\infty} A_k$, then $\lim_{k\to\infty} P(A_k) = P(A)$.

${}_nP_k = \frac{n!}{(n-k)!}$ $\qquad$ $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$ $\qquad$ $\binom{n}{n_1 \ \cdots \ k_\ell} = \frac{n!}{k_1! \ \cdots \ k_\ell!}$

$P(B|A) \overset{def}{=} \frac{P(A\cap B)}{P(A)}$ $\qquad$ $P(A \cap B) = P(A)P(B|A)$ $\qquad$ $P(B) = \sum_{k=1}^{\infty} P(B|A_k)P(A_k)$

$P(A_j|B) = \frac{P(A_j)P(B|A_j)}{\sum_{k=1}^{\infty} P(A_k)P(B|A_k)}$ $\qquad$ $P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A)+P(A^c)P(B|A^c)}$

A and B independent means $P(A \cap B) = P(A)P(B)$ $\qquad$ $P(k \text{ heads}) = \binom{n}{k}\theta^k(1-\theta)^{n-k}$

$F_X(x) = P(X \le x)$ $\qquad$ $\lim_{x\to-\infty} F(x) = 0$ $\qquad$ $\lim_{x\to\infty} F(x) = 1$

If X is continuous, $\qquad$ $P(X \in A) = \int_A f_X(x)\, dx$ $\qquad$ $\frac{d}{dx} F_X(x) = f_X(x)$

If X is discrete, $\qquad$ $p_X(x) \overset{def}{=} P(X = x)$

$F_{X,Y}(x,y) = P(X \le x, Y \le y)$ $\qquad$ $\lim_{x\to\infty} F_{X,Y}(x,y) = F_Y(y)$ $\qquad$ $\lim_{y\to\infty} F_{X,Y}(x,y) = F_X(x)$

If X and Y are discrete, $\qquad$ $p_{X,Y}(x,y) \overset{def}{=} P(X = x, Y = y)$ $\qquad$ $p_X(x) = \sum_y p_{X,Y}(x,y)$

If X and Y are continuous, $\qquad$ $P\{(X,Y) \in A\} = \iint_A f_{X,Y}(x,y)\, dx\, dy$ $\qquad$ $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy$

$\frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y) = f_{X,Y}(x,y)$

$p_{Y|X}(y|x) \stackrel{def}{=} \frac{p_{X,Y}(x,y)}{p_X(x)}$ $\qquad$ $f_{Y|X}(y|x) \stackrel{def}{=} \frac{f_{X,Y}(x,y)}{f_X(x)}$

Independence: $F_{X,Y}(x,y) = F_X(x)F_Y(y) \quad \Leftrightarrow \quad p_{X,Y}(x,y) = p_X(x)\,p_Y(y)$ or $f_{X,Y}(x,y) = f_X(x)\,f_Y(y)$

Convolution formulas: If X and Y are independent random variables, and $Z = X + Y$

$p_Z(z) = \sum_x p_X(x)p_Y(z-x)$ $\qquad$ $f_z(z) = \int_{-\infty}^{\infty} f_X(x)f_Y(z-x)\,dx$

Jacobian formula: $Y_1 = g_1(X_1, X_2)$ and $Y_2 = g_2(X_1, X_2)$

$$f_{Y_1,Y_2}(y_1,y_2) = f_{X_1,X_2}(\,x_1(y_1,y_2), x_2(y_1,y_2)\,) \cdot abs \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix}$$

$$f_{Y_1,Y_2}(y_1,y_2) = f_{x_1x_2}(\,x_1(y_1,y_2), x_2(y_1,y_2)\,) \cdot abs\left(\frac{\partial x_1}{\partial y_1}\frac{\partial x_2}{\partial y_2} - \frac{\partial x_1}{\partial y_2}\frac{\partial x_2}{\partial y_1}\right)$$

Change to polar coordinates: $dx\,dy = r\,dr\,d\theta$

$E(X) \stackrel{def}{=} \sum_x x\,p_X(x)$ or $\int_{-\infty}^{\infty} x\,f_X(x)\,dx$ $\qquad$ $E(g(X)) = \sum_x g(x)\,p_X(x)$ or $\int_{-\infty}^{\infty} g(x)\,f_X(x)\,dx$

$E\left(\sum_{i=1}^n a_iX_i\right) = \sum_{i=1}^n a_iE(X_i)$ $\qquad$ $E(X) = E(E[X|Y])$

$Var(X) \stackrel{def}{=} E\left((X-\mu)^2\right)$ $\qquad$ $Var(X) = E(X^2) - [E(X)]^2$

$Var(a+bX) = b^2Var(X)$ $\qquad$ $Var(aX+bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X,Y)$

$Cov(X,Y) \stackrel{def}{=} E[(X-\mu_X)(Y-\mu_Y)]$ $\qquad$ $Cov(X,Y) = E(XY) - E(X)E(Y)$

$Cov(a+bX, c+dY) = bd\,Cov(X,Y)$ $\qquad$ $Cov(X, aY+bZ) = a\,Cov(X,Y) + b\,Cov(X,Z)$

$Var\left(\sum_{i=1}^n a_iX_i\right) = \sum_{i=1}^n a_i^2Var(X_i) \; + \; \sum\sum_{i\neq j} a_ib_jCov(X_i, X_j)$

Markov's inequality $\qquad$ *Chebyshev's inequality*

If $P(Y \geq 0) = 1$, then $E(Y) \geq a\,P(Y \geq a)$ $\qquad$ $P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$

$M_X(t) \stackrel{def}{=} E(e^{Xt})$ $\qquad$ $M_X^{(k)}(0) = E(X^k)$

$M_{aX}(t) = M_X(at)$ $\qquad$ $M_{\sum X_i}(t) = \prod_{i=1}^n M_{X_i}(t)$ if the X_i are independent.

Convergence in probability:

$T_n \stackrel{p}{\to} c$ means for all $\epsilon > 0$, $\lim\limits_{n\to\infty} P\{|T_n - c| \geq \epsilon\} = 0 \Leftrightarrow \lim\limits_{n\to\infty} P\{|T_n - c| < \epsilon\} = 1$

Variance rule: If $\lim\limits_{n\to\infty} E(T_n) = c$ and $\lim\limits_{n\to\infty} Var(T_n) = 0$, then $T_n \stackrel{p}{\to} c$.

Law of Large Numbers: $\overline{X}_n \stackrel{p}{\to} \mu = E(X_i)$.

Continuous mapping: If $T_n \stackrel{p}{\to} c$ and $g(x)$ is continuous at $x = c$, then $g(T_n) \stackrel{p}{\to} g(c)$

Convergence in Distribution:

$X_n \stackrel{d}{\to} X$ means $\lim\limits_{n\to\infty} F_{X_n}(x) = F_X(x)$ at every point where $F_X(x)$ is continuous.

Central Limit Theorem: $Z_n = \frac{\sqrt{n}(\overline{X}_n - \mu)}{\sigma} \stackrel{d}{\to} Z \sim$ Normal (0,1).

$T_n \stackrel{d}{\to} c \Leftrightarrow T_n \stackrel{p}{\to} c$.

Distribution	**Density or probability mass function**	**MGF** $M_X(t)$	$E(X)$	$Var(X)$
Bernoulli (θ)	$p_X(x) = \theta^x(1-\theta)^{1-x}$ for $x = 0, 1$	$\theta e^t + 1 - \theta$	θ	$\theta(1-\theta)$
Binomial (n, θ)	$p_X(x) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$ for $x = 0, \ldots, n$	$(\theta e^t + 1 - \theta)^n$	$n\theta$	$n\theta(1-\theta)$
Geometric (θ)	$p_X(x) = (1-\theta)^x\,\theta$ for $x = 0, 1, 2, \ldots$	$\theta\left(1-(1-\theta)e^t\right)^{-1}$	$\frac{1-\theta}{\theta}$	$\frac{1-\theta}{\theta^2}$
Negative Binomial (r, θ)	$\binom{x+r-1}{x}\theta^r\,(1-\theta)^x$ for $x = 0, 1, \ldots$			
Hypergeometric (N, M, n)	$p_X(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$, where $\binom{a}{b}$ must make sense.			
Poisson (λ)	$p_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ for $x = 0, 1, \ldots$	$e^{\lambda(e^t-1)}$	λ	λ
Multinomial $(n, \theta_1, \ldots, \theta_r)$	$p_{\mathbf{X}}(x_1, \ldots, x_r) = \binom{n}{n_1 \cdots n_r}\theta_1^{x_1}\cdots\theta_r^{x_r}$			
Uniform (L, R)	$f_X(x) = \frac{1}{R-L}$ for $L \leq x \leq R$	$\frac{e^{Rt}-e^{Lt}}{t(R-L)}$ for $t \neq 0$	$\frac{R-L}{2}$	$\frac{(R-L)^2}{12}$
Exponential (λ)	$f_X(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ for $x \geq 0$	$(1-\frac{t}{\lambda})^{-1}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma (α, λ)	$f_X(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)}e^{-\lambda x}\,x^{\alpha-1}$ for $x \geq 0$	$(1-\frac{t}{\lambda})^{-\alpha}$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$
Normal (μ, σ^2)	$\frac{1}{\sigma\sqrt{2\pi}}\exp -\left\{\frac{(x-\mu)^2}{2\sigma^2}\right\}$	$e^{\mu t+\frac{1}{2}\sigma^2t^2}$	μ	σ^2
Beta	$f_X(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\,x^{\alpha-1}(1-x)^{\beta-1}$ for $0 \leq x \leq 1$		$\frac{\alpha}{\alpha+\beta}$	

If $X \sim$ Exponential (λ), $F_X(x) = 1 - e^{-\lambda x}$. If $X \sim$ Normal (μ, σ^2), $\frac{X-\mu}{\sigma} \sim$ Normal(0,1)

This formula sheet was prepared by Jerry Brunner, Department of Mathematical and Computational Sciences, University of Toronto Mississauga. It is licensed under a Creative Commons Attribution - ShareAlike 3.0 Unported License. Use any part of it as you like and share the result freely. The LaTeX source code is available from the course website:

http://www.utstat.toronto.edu/~brunner/oldclass/256f19

D.2 Standard Normal Cdf

If $Z \sim N(0, 1)$, then we can use Table D.2 to compute the cumulative distribution function (cdf) Φ for Z. For example, suppose we want to compute $\Phi(z) = P(Z < 1.03)$. The symmetry of the $N(0, 1)$ distribution about 0 implies that $\Phi(z) = 1 - \Phi(-z)$, so using Table D.2, we have that $P(Z < 1.03) = P(Z < 1.03) = 1 - P(Z < -1.03) = 1 - 0.1515 = 0.8485$.

Table D.2 Standard Normal Cdf

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641