STA 256f19 Assignment Five[1]

Please read Section 2.4 and the first part of 2.5 (pages 51-68) in the text. You can skip 2.5.4 and 2.5.5. Also, look over your lecture notes. The following homework problems are not to be handed in. They are preparation for Term Test 2 and the final exam. Use the formula sheet.

1. Do Exercise 2.4.1 in the text.
2. Let $X \sim$ Exponential(λ).
   (a) Find the cumulative distribution function $F_X(x)$. Be sure it is defined for all real x.
   (b) Do Exercise 2.4.3, parts a-c only. The answer to (d) in the book is incorrect
3. Do Exercise 2.4.5 in the text.
4. Do Exercise 2.4.7 in the text.
5. Do Exercise 2.4.9 in the text.
6. Do Exercise 2.4.10 in the text.
7. Do Exercise 2.4.14 through 2.4.19 in the text.
8. Do Exercise 2.4.21 in the text. "Recall" that $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$. The Cauchy distribution is the problem child of Statistics. Frequently, results that seem to be true in general are not true for the Cauchy. This is useful because it helps us recognize the limitations of our knowledge.
9. The continuous random variable X has density $f_X(x) = \left\{ \begin{array}{ll} \frac{\alpha}{x^{\alpha+1}} & \mbox{for } x \geq 1 \\ 0 & \mbox{for } x < 1 \end{array} \right.$, where $\alpha > 0$.
   Let $Y = 1/X$. Find the density of Y. Be careful to specify where it is non-zero. If you look at it carefully, you will see that this is a beta distribution with $\beta = 1$.
10. Let the continuous random variable X have cumulative distribution function $F(x) = \left\{ \begin{array}{ll} 0 & \mbox{for } x < 0 \\ x^3 & \mbox{for } 0 \leq x \leq 1 \\ 1 & \mbox{for } x > 1 \end{array} \right.$.
    Find the density $f(x)$. Be careful to specify where it is non-zero.
11. The density of the normal distribution with parameters μ and σ^2 is given on the formula sheet.
    (a) Show that $f(x)$ is symmetric about μ, meaning $f(\mu + x) = f(\mu - x)$.
    (b) Let $X \sim$ N(μ, σ) and $Z = \frac{X-\mu}{\sigma}$. Find the density of Z. Where is it non-zero?
    (c) Let $X \sim$ N(μ, σ) and $Y = aX + b$, where a and b are constants, and $a \neq 0$. Find the density of Y. Where is it non-zero? Do you recognize this density?
    (d) Let $Z \sim N(0, 1)$. This is the standard normal, in which $\mu = 0$ and $\sigma^2 = 1$. If $x > 0$, show $F_Z(-x) = 1 - F_Z(x)$. Hint: Write $F_Z(-x)$ as an integral and do a change of variables.
    (e) Let $X \sim N(\mu = 50, \sigma^2 = 100)$. For the following, use Table D2 on page 712. It will be provided with the test and final exam. I suggest drawing pictures.

[1]Copyright information is at the end of the last page.

i. Find $P(X < 60)$. [0.8413]

ii. Find $P(X > 30)$. [0.9772]

iii. Find $P(30 < X < 55)$. [0.6687]

(f) Let $Z \sim N(0,1)$, the standard normal. Show that the standard normal density integrates to one. Hint: Split the integral at zero and let $t = \frac{z^2}{2}$. You may use $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ for now without proof.

(g) Using the last result, show that the general normal density integrates to one.

(h) Let $Z \sim N(0,1)$ and $Y = Z^2$.

i. For what values of y is $f_y(y) > 0$?

ii. Show that Y has a gamma distribution and give the parameters. Again, you may use the fact that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$, without proof.

12. Let X have an Exponential(λ) distribution, and let $Y = \lambda X$. Find the density of Y. Be sure to specify where it is non-zero.

13. Let the continuous random variable X have distribution function $F_{_X}(x)$, and let $Y = F_{_X}(X)$. That's right. You are transforming a random variable by its own cumulative distribution function.

(a) For what values of y is $f_{_Y}(y) > 0$?

(b) Find $f_{_Y}(y)$. Do you recognize this distribution?

14. Let the continuous random variable X have cumulative distribution function $F_{_X}(x)$ and density $f_{_X}(x)$. The distribution function is strictly increasing on a single interval (which could be infinite), so that the inverse function $F_{_X}^{-1}(y)$ is defined in the natural way. Let $Y = F_{_X}^{-1}(U)$, where U is a continuous uniform random variable on the interval from zero to one. Find the cumulative distribution function and density of Y.
