# Assignment 3

(3) 15R 5B

$$P(R_1 \cap B_2) = P(R_1)\,P(B_2 \mid R_1)$$
$$= \frac{15}{20} \cdot \frac{5}{19} = \frac{15}{76} = 0.197$$

(5) 3R 4W $\to 2$

(a) $P(W_2 \mid R_1) = \frac{4}{6}$

(b) $$P(W_2) = P(R_1)P(W_2 \mid R_1) + P(W_1)P(W_2 \mid W_1)$$
$$= 3/7 \cdot \frac{4}{6} + \frac{4}{7} \cdot \frac{1}{2}$$
$$= \frac{2}{7} + \frac{2}{7} = \frac{4}{7}$$

(c) $$P(W_1 \mid W_2) = \frac{P(W_1 \cap W_2)}{P(W_2)} = \frac{P(W_1)\,P(W_2 \mid W_1)}{P(W_2)}$$
$$= \frac{\frac{2}{7}}{\frac{2}{7} + \frac{2}{7}} = \frac{1}{2}$$

(6) $$P(A_1)\,P(A_2 \mid A_1)\,P(A_3 \mid A_1 \cap A_2)\,P(A_4 \mid A_1 \cap A_2 \cap A_3)$$
$$= \cancel{P(A_1)} \cdot \frac{\cancel{P(A_1 \cap A_2)}}{\cancel{P(A_1)}} \cdot \frac{\cancel{P(A_1 \cap A_2 \cap A_3)}}{\cancel{P(A_1 \cap A_2)}} \cdot \frac{P(A_1 \cap A_2 \cap A_3 \cap A_4)}{\cancel{P(A_1 \cap A_2 \cap A_3)}}$$
$$= P(A_1 \cap A_2 \cap A_3 \cap A_4)$$

9 $A = H$ (3 R, 2 W) $\qquad$ $B = T$ (2 R, 5 W)

(a) $$P(R) = P(H)P(R \mid H) + P(T)P(R \mid T)$$
$$= \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{2}{7}$$
$$= \frac{3}{10} + \frac{1}{7} = \frac{21+10}{70} = \frac{31}{70}$$

(b) $$P(H \mid R) = \frac{P(H \cap R)}{P(R)} = \frac{P(H)P(H \mid R)}{P(R)}$$

$$= \frac{21/70}{31/70} = \frac{21}{31}$$

10 (a) $$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \geq 0$$ since both numerator & denominator are non-negative

$$(A \cap B) \subseteq A \implies P(A \cap B) \leq P(A) \implies \frac{P(A \cap B)}{P(A)} \leq 1$$

(b) $$P(\emptyset \mid A) = \frac{P(\emptyset \cap A)}{P(A)} = \frac{P(\emptyset)}{P(A)} = \frac{0}{P(A)} = 0$$

(c) $$P(A \mid A) = \frac{P(A \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

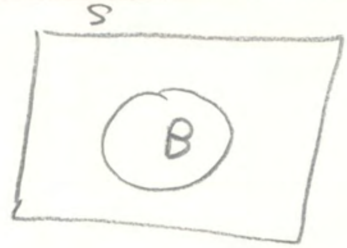
c.

(10d)

| | | |
|---|---|---|
| 1. | $A \cap \bigcup_{k=1}^{\infty} B_k = \bigcup_{k=1}^{\infty} (A \cap B_k)$ | Distributive Law |
| 2. | And the $(A \cap B_k)$ are disjoint | Because the $B_k$ are disjoint |
| 3. | So $P(\bigcup_{k=1}^{\infty} A \cap B_k) = \sum_{k=1}^{\infty} P(A \cap B_k)$ | Property 4 |
| 4. | Now $P(\bigcup_{k=1}^{\infty} B_k \mid A) = \dfrac{P(A \cap \bigcup_{k=1}^{\infty} B_k)}{P(A)}$ | Definition |
| 5. | $= \dfrac{1}{P(A)} \sum_{k=1}^{\infty} P(A \cap B_k)$ | step 3 |
| 6. | $= \sum_{k=1}^{\infty} \dfrac{P(A \cap B_k)}{P(A)}$ | Math |
| 7. | $= \sum_{k=1}^{\infty} P(B_k \mid A)$ | Definition |

d.

13 (a)

| | |
|---|---|
| $B = B \cap S$ |  |
| $= B \cap \bigcup_{k=1}^{\infty} A_k$ | Given |
| $= \bigcup_{k=1}^{\infty} (A_k \cap B)$ | Distributive Law |
| disjoint | Because the $A_k$ are disjoint |
| So $P(B) = \sum_{k=1}^{\infty} P(A_k \cap B)$ | Property 4 |
| $= \sum_{k=1}^{\infty} P(A_k) \dfrac{P(A_k \cap B)}{P(A_k)}$ | Math |
| $= \sum_{k=1}^{\infty} P(A_k) P(B \mid A_k)$ | Definition |

(b) $P(B_j \mid A) = \dfrac{P(A \cap B_j)}{P(A)} = \dfrac{P(B_j)}{P(A)}$

$=$

(14) $\overset{\nearrow 2}{\begin{matrix}8R\\4W\end{matrix}}$ $P(W_1|R_2) = \dfrac{P(W_1 \cap R_2)}{P(R_2)}$

$$= \frac{P(R_2|W_1)P(W_1)}{P(W_1)P(R_2|W_1) + P(R_1)P(R_2|R_1)}$$

$$= \frac{\frac{8}{\cancel{11}} \cdot \frac{4}{\cancel{12}}}{\frac{4}{\cancel{12}} \frac{8}{\cancel{11}} + \frac{8}{\cancel{12}} \frac{7}{\cancel{11}}} = \frac{8 \cdot 4}{8(4+7)} = \frac{4}{11}$$

(15) $P(DS) = \frac{1}{940}$

$P(T|D) = \frac{84}{100}$, $P(T|D^c) = \frac{4}{100}$

$$P(D|T) = \frac{P(D \cap T)}{P(T)} = \frac{P(D)P(T|D)}{P(D)P(T|D) + P(D^c)P(T|D^c)}$$

$$= \frac{\frac{1}{\cancel{940}} \cdot \frac{84}{\cancel{100}}}{\frac{1}{\cancel{940}} \cdot \frac{84}{\cancel{100}} + \frac{939}{\cancel{940}} \cdot \frac{4}{\cancel{100}}} = \frac{84}{84 + 3752}$$

$$= \frac{84}{3836} = \frac{2 \cdot 42}{2 \cdot 1918} = \frac{2 \cdot 21}{2 \cdot 959}$$

$$= \frac{3}{137}$$

(18) $P(\text{At least once}) = 1 - P(\text{All miss})$

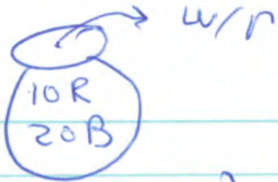
$= 1 - .9^n \overset{set}{=} 0.6 \iff .9^n = 0.4$

$\iff n \log .9 = \log .4 \Rightarrow n = \frac{\ln .4}{\ln .9} = 8.7$ so

let $n = 9$ $\quad 1 - .9^8 = 0.569$

$1 - .9^9 \approx 0.612$

f.

(19) 


(a) $P(BRBBR) = \left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3 = \frac{8}{3^5} \approx 0.0329$

(b) $\binom{5}{2} = 10$

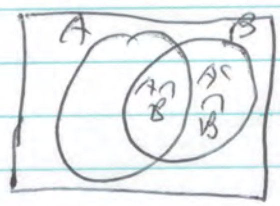
(c) $\binom{5}{2}\frac{8}{3^5} \approx 0.329$

(d) $\binom{5}{j}\left(\frac{1}{3}\right)^j\left(\frac{2}{3}\right)^{5-j}$

(20) $\binom{n}{k}\theta^k(1-\theta)^{n-k}$

(21) 1.5.14 says prove $A$ & $B$ ind iff $A^c$ & $B$ ind

$B = (A\cap B)\cup(A^c\cap B)$
disjoint, so




$P(B) = P(A\cap B) + P(A^c\cap B)$
by Prop 4

Spse $A$ & $B$ are independent

$$P(A^c\cap B) = P(B) - P(A\cap B) = P(B) - P(A)P(B)$$
$$= P(B)(1-P(A)) = P(A^c)P(B),\text{ and}$$

$A^c$ & $B$ are independent. Spse $A^c$ & $B$ are ind.

$$P(A\cap B) = P(B) - P(A^c\cap B) = P(B) - P(A^c)P(B)$$
$$= P(B)(1-P(A^c)) = P(A)P(B),\text{ and}$$

$A$ & $B$ are independent

■

(22) Choose (d). If $A$ & $B$ are disjoint, $A \cap B = \emptyset$ and $P(A \cap B) = P(\emptyset) = 0 \neq P(A)P(B)$, since $P(A)$ & $P(B)$ are both positive.

(23) If $P(A) = 0$, then $A \cap B \subseteq A$ so

$0 \leq P(A \cap B) \leq P(A) = 0$, so $P(A \cap B) = 0$

$0 = P(A \cap B) = 0 \cdot P(B) = P(A)P(B)$. That is, $A$ & $B$ are independent.

(24) If $P(B) = 1$ it is ~~possible~~ guaranteed, because

$$P(A \cap B) = P(A) = P(A) \cdot 1 = P(A)P(B)$$

If $P(B) < 1 \Rightarrow P(A)P(B) < P(A)$

Then $P(A \cap B) = P(A) > P(A)P(B)$, and $A$ & $B$ are not independent

25 (a) $\left(\frac{5}{6}\right)^3 \cdot \frac{1}{6}$

(b) $\sum_{k=1}^{\infty}\left(\frac{5}{6}\right)^{k-1} \cdot \frac{1}{6} = \frac{1/6}{5/6} \sum_{k=1}^{\infty}\left(\frac{5}{6}\right)^{k}$ Geometric sum

$= \frac{1}{5} \frac{5/6}{1-5/6} = \frac{1}{5} \cdot \frac{5/6}{1/6} = 1$

(c) First 2 occurs on roll 1 or 3 or 5 or ...

$\theta + (1-\theta)^2\theta + (1-\theta)^4\theta + \cdots$

$= \theta \sum_{k=0}^{\infty}(1-\theta)^{2k} = \theta \sum_{k=0}^{\infty}\left[(1-\theta)^2\right]^k$

$= \theta \cdot \frac{((1-\theta)^2)^0}{1-(1-\theta)^2} = \frac{\theta}{1-(\theta^2-2\theta+1)} = \frac{\theta}{\cancel{1}-\theta^2+2\theta-\cancel{1}}$

$= \frac{\theta}{\theta(2-\theta)} = \frac{1}{2-\theta} = \frac{1}{2-1/6} = \frac{1}{\frac{12}{6}-\frac{1}{6}}$

$= 6/11$

(d) $\binom{4}{2}\theta^2(1-\theta)^2 \cdot \theta$ with $\theta = \frac{1}{6}$