Assignment 2

16. 8 children standing in line

a) $8! = 40,320$

b) $7! \times 2 = 5040 \times 2 = 10,080$

c) $4!\,4! \times 2 = 1152$

d) $\dfrac{4!\,4! \times 2}{8!} = \dfrac{1152}{40320} \approx 0.02857$

17. $\binom{52}{13\ 13\ 13\ 13}$

18. There are $52-20=32$ cards in an "other" pile.

$\binom{52}{5\ 5\ 5\ 5\ 32}$ straight from lecture

19. $\dfrac{\binom{10}{2}\binom{15}{5}}{\binom{25}{7}}$ Cream filled from final

20. 7 Black socks, 8 blue, 9 green. Pick 2

a) P(Match) $= \dfrac{\binom{7}{2}+\binom{8}{2}+\binom{9}{2}}{\binom{24}{2}}$

b) P(Black pair) $= \binom{7}{2} \Big/ \binom{24}{2}$

Exercise for section 1.4

1.4.1 a) $1/6^8$

b) $6/6^8 = 1/6^7$

c) Seven ones & a two. There are 8 ways to choose the die with a two

$$8/6^8$$

1.4.2 This is a binomial problem before independence. Using (1.4.2)

$$\binom{10}{2}\left(\frac{5}{6}\right)^8\left(\frac{1}{6}\right)^2 = \binom{10}{2}\frac{5^8}{6^{10}} \approx 0.2907$$

1.4.3 Using the expression on p. 17 before (1.4.2),

$$P(k \text{ Heads}) = \binom{100}{k}\frac{1}{2^{100}}$$

Answer is

$$1 - \left[\binom{100}{0}\frac{1}{2^{100}} + \binom{100}{1}\frac{1}{2^{100}} + \binom{100}{2}\frac{1}{2^{100}}\right]$$

$$= 1 - \frac{1}{2^{100}}\left(1 + 100 + \frac{100!}{2!\,98!}\right)$$

$$= 1 - \frac{1}{2^{100}}\left(1 + 100 + \frac{100\cdot 99}{2}\right)$$

$$= 1 - \frac{1}{2^{100}}\left(1 + 100 + 50\cdot 99\right)$$

$$= 1 - \frac{5051}{2^{100}}$$ Answer in Book ≈ 1

1.4.4 (a) $\dfrac{1}{\binom{52}{5}}$ 4 aces + K of Spades

(b) $\binom{13}{5} \Big/ \binom{52}{5}$ All 5 spades

(c) No pairs (all 5 diff values)

$$\frac{\binom{13}{5} \cdot 4^5}{\binom{52}{5}}$$

Choose the values, & then for each there are 4 suits

$$= \frac{1287 \cdot 1024}{\binom{52}{5}} = \frac{1317888}{2\,598\,960}$$

≈ 0.50708

(d) Full House

$$\frac{\binom{13}{1} \cdot \binom{4}{3} \cdot \binom{12}{1} \cdot \binom{4}{2}}{\binom{52}{5}} = \frac{13 \cdot 4 \cdot 12 \cdot 6}{\binom{52}{5}}$$

$$= \frac{3{,}744}{2{,}598{,}960} \approx 0.0014405$$

More ways to get (c)

$$\frac{52 \cdot 48 \cdot 44 \cdot 40 \cdot 36}{5!} \Big/ \binom{52}{5} = 1 \cdot \frac{48}{51} \cdot \frac{44}{50} \cdot \frac{40}{49} \cdot \frac{36}{48}$$

TREE

Drop 3 cards on the ground each time

(1.4.7) Deal cards until 1st Jack. want prob at least 10 cards go by before 1st jack. That's deal 10 cards. want prob of no jacks

Denominator is $_{52}P_{10} = \dfrac{52!}{42!} = 52 \cdot 51 \cdots 43$

Numerator is # of ways to deal 10 from 48 non-jack

$$\text{Prob} = \frac{_{48}P_{10}}{_{52}P_{10}} = \frac{48!/38!}{52!/42!}$$

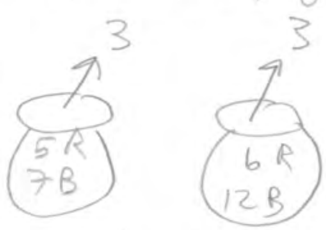
$$= \frac{48!\,42!}{52!\,38!} = \frac{\cancel{48!}\,42\cdot 41\cdot 40\cdot 39\cdot \cancel{38!}}{52\cdot 51\cdot 50\cdot 49\cdot \cancel{48!}\,\cancel{38!}}$$

$$= \frac{42\cdot 41\cdot 40\cdot 39}{52\cdot 51\cdot 50\cdot 49}$$

Book has $\binom{48}{10} \Big/ \binom{52}{10}$ which is just a 10! in NUM & denom.

(1.4.9) Independence, geometric, later

(1.4.11)



P(All 6 same colour)
= P(6 Red) + P(6 Blue)

$$\frac{\binom{5}{3}\binom{6}{3}}{\binom{12}{3}\binom{18}{3}} + \frac{\binom{7}{3}\binom{12}{3}}{\binom{12}{3}\binom{18}{3}}$$

Checks with the book. Book's grouping suggests use of independence