STA 256f19 Assignment Two[1]

Please read Sections 1.2, 1.3 and 1.4 in the text (pages 4-20) and review lecture sets entitled *Sets*, *Foundations of Probability* and *Counting*. These homework problems are not to be handed in. They are preparation for Term Test 1 and the final exam. Use the formula sheet.

1. Do Exercises 1.2.6 and 1.2.7 in the text. For 1.2.6, just give regions a, b and e. The answers are $a = A \cap B^c \cap C^c$, $b = A \cap B \cap C^c$, and $e = A \cap B \cap C$.

2. Make Venn diagrams to illustrate the distributive laws:

   (a) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

   (b) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

3. Make Venn diagrams to illustrate the De Morgan laws:

   (a) $(A \cap B)^c = A^c \cup B^c$

   (b) $(A \cup B)^c = A^c \cap B^c$

4. Make a Venn diagram showing that if A and B are disjoint, then $A \cap C$ and $B \cap C$ are also disjoint.

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5. Prove Property 5: $P(A^c) = 1 - P(A)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

6. Prove Property 6: If $A \subseteq B$ then $P(A) \leq P(B)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

7. Prove Property 7 (the inclusion-exclusion principle): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

8. Do Exercises 1.2.1, 1.2.3 and 1.2.4 in the text. The answer to 1.2.4 is No. Consider $P(\{2, 3\})$.

9. Do problems 1.2.13, 1.2.14 and 1.2.15. The answers are No, No, Yes. Here is some background information you may not have seen yet. An infinite set is said to be *countable* if it can be placed in a one-to-one correspondence with the set of natural numbers $1, 2, \ldots$. By Cantor's famous diagonalization proof, the real numbers between zero and one are not countable. Thus, adding up their probabilities is not possible.

10. Let $A_1, A_2, \ldots$ form a partition of the sample space S, meaning that $A_1, A_2, \ldots$ are disjoint and $S = \cup_{k=1}^{\infty} A_k$. Let B be any event. Show that $P(B) = \sum_{k=1}^{\infty} P(A_k \cap B)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

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11. Let $A_1, A_2, \ldots$ be a collection of events, not necessarily disjoint. Show that $P\left(\cup_{k=1}^{\infty} A_k\right) \leq \sum_{k=1}^{\infty} P(A_k)$. Use the Properties 1-7 of probability and the tabular format illustrated in lecture.

12. Do Exercises 1.3.1, 1.3.3 and 1.3.5. For Exercise 1.3.5, what is the sample space? Assume all outcomes are equally likely.

13. Do Problem 1.3.9 in the text. It may be helpful to make a Venn diagram. The answer is that $P(\{2\})$ could be as small as zero, and as large as 0.2.

14. Let $A_1 \subseteq A_2 \subseteq A_3 \subseteq \ldots$ and let $A = \cup_{k=1}^{\infty} A_k$. Show that $\lim_{k\to\infty} P(A_k) = P(A)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

15. Let $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots$ and let $A = \cap_{k=1}^{\infty} A_k$. Show that $\lim_{k\to\infty} P(A_k) = P(A)$. Use Properties 1-4 of probability and the tabular format illustrated in lecture.

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16. If eight children are standing in line,
    (a) In how many orders can they stand?
    (b) In how many orders can they stand if two friends insist on being together?
    (c) Suppose there are four boys and four girls. In how many orders can they stand if the boys stay together and the girls stay together?
    (d) If the children line up completely at random, what is the probability that the four boys are together and the four girls are together?

17. The four players in a bridge game are each dealt 13 cards from an ordinary 52-card deck. How many ways are there to do this? The answer has 29 digits, so stop simplifying after you arrive at a multinomial coefficient.

18. In a game of poker, four players are each dealt 5 cards from a 52-card deck. How many ways are there to deal the cards?

19. A box of 25 Valentine's Day chocolates has 10 that are cream filled and 15 that are not cream filled. If you eat 7 chocolates at random, what is the probability that you get exactly 2 cream filled? Just write down the answer. There is no need to simplify. This question was on the 2018 final exam.

20. This question is taken from *Mathematical statistics and data analysis*, by Rice. A drawer of socks contains seven black socks, eight blue socks, and nine green socks. Two socks are chosen in the dark.
    (a) What is the probability that they match?
    (b) What is the probability that a black pair is chosen?

21. Do Exercise 1.4.1 in the text.

22. Do Exercise 1.4.4 in the text.

23. Do Exercise 1.4.5 in the text.

24. Do Exercise 1.4.7 in the text.

25. Do Exercise 1.4.11 in the text. The sampling is without replacement.

26. A jar contains 10 red balls and 20 blue balls. If you sample 5 balls randomly without replacement, what is the probability of

    (a) All blue?

    (b) Two red and three blue?

    (c) At least one red?

    (d) Obtaining j red balls, $j = 0, \ldots, 5$? Give a single formula. Don't simplify.

27. Do Challenge problem 1.4.21 in the text. The answers are not in the back of the book. They are

    (a) $\frac{365 \cdot 364}{365^2}$

    (b) $\frac{365}{365^C}$

    (c) $1 - \frac{365 \cdot 364 \cdots (365-C+1)}{365^C} = 1 - \frac{{}_{365}P_C}{365^C} = 1 - \frac{365!}{(365-C)! 365^C}$

    (d) $C = 23$. I had to write a program to find this, so you could not be expected to do it on a test or the final exam.

Here are some more answers that are not in the back of the book.

16. (a) $8!$
    (b) $7! \cdot 2$
    (c) $4!4! \cdot 2$
    (d) $\frac{4!4! \cdot 2}{8!} \approx 0.02857$
17. $\binom{52}{13\ 13\ 13\ 13}$
18. $\binom{52}{5\ 5\ 5\ 5\ 32}$
19. $\frac{\binom{10}{2}\binom{15}{5}}{\binom{25}{7}}$
20. (a) $\frac{\binom{7}{2}+\binom{8}{2}+\binom{9}{2}}{\binom{24}{2}}$
    (b) $\frac{\binom{7}{2}}{\binom{24}{2}}$
