Name Jerry

Student Number

STA 256 f2019 LEC0101 Test 2

Tutorial Section (Circle One)

TUT0101 Mon. 3-4 DH 2080 Ali	TUT0102 Mon. 4-5 DH 2080 Dashvin	TUT0103 Mon. 5-6 IB 360 Dashvin	TUT0104 Mon. 6-7 IB 240 Ali	TUT0105 Wed. 4-5 IB360 Marie
TUT0106 Wed. 5-6 IB 360 Marie	TUT0107 Fri. 9-10 IB 200 Crendall	TUT0108 Fri. 10-11 DH 2070 Ali	TUT0109 Fri. 10-11 DV 3093 Cendall	TUT0110 Fri. 11-12 DH 2070 Ali
TUT0111 Fri. 11-12 DV 3093 Crendall	TUT0112 Fri. 12-1 DV 2070 Crendall	TUT0113 Fri. 4-5 DV 3093 Karan	TUT0114 Fri. 5-6 IB 360 Karan	TUT0115 Fri. 6-7 IB 360 Karan
TUT0116 Wed. 11-12 DH 2070 Ana	TUT0117 Wed. 12-1 IB 260 Ana			

Question	Value	Score
1-5		
6		
7		
8		
9		
10		
Total = 100 Points		

CBBDA

Circle the alternative that is *closest* to the correct answer.

1. (5 points) Let X have a normal distribution with parameters $\mu = 100$ and $\sigma^2 = 400$. What is $P(60 < X < 140)$? Circle the letter.

   (a) 0.0228
   (b) 0.0456
   (c) 0.9544
   (d) 0.9772

   $P(60 < X < 140) = P\left(\frac{60-100}{20} < \frac{X-\mu}{\sigma} < \frac{140-100}{20}\right)$

   $= P(-2 < Z < 2) = 1 - 2(0.0228)$

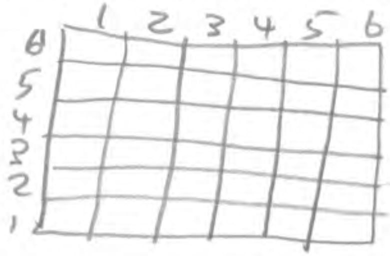
2. (5 points) Let X have a continuous Uniform(0,1) distribution. What is $P(-1/4 < X < 1/4)$? Circle the letter.

   (a) 0.00
   (b) 0.25
   (c) 0.50
   (d) 0.75

3. (5 points) Roll two fair dice, one red and one green. Let X be the number showing on the red die and Y be the number showing on the green die. What is $F_{X,Y}(8,2)$? Circle the letter.

   (a) 0.000
   (b) 0.333
   (c) 0.667
   (d) 1.000

   8

4. (5 points) Let X have a Geometric distribution with parameter $\theta = 1/4$. What is $F_X(5)$? Circle the letter.

   (a) 0.1780
   (b) 0.2373
   (c) 0.7627
   (d) 0.8220

   $1 - \sum_{x=6}^{\infty} \left(1-\frac{1}{4}\right)^x \left(\frac{1}{4}\right) = 1 - \frac{1}{4} \sum_{x=6}^{\infty} \left(\frac{3}{4}\right)^x$

   $= 1 - \frac{1}{4} \frac{(3/4)^6}{\cancel{1-3/4}} = 1 - \left(\frac{3}{4}\right)^6$

5. (5 points) Let Z have a Standard Normal distribution; that is, $\mu = 0$ and $\sigma^2 = 1$. What is $P(Z = -2)$? Circle the letter.

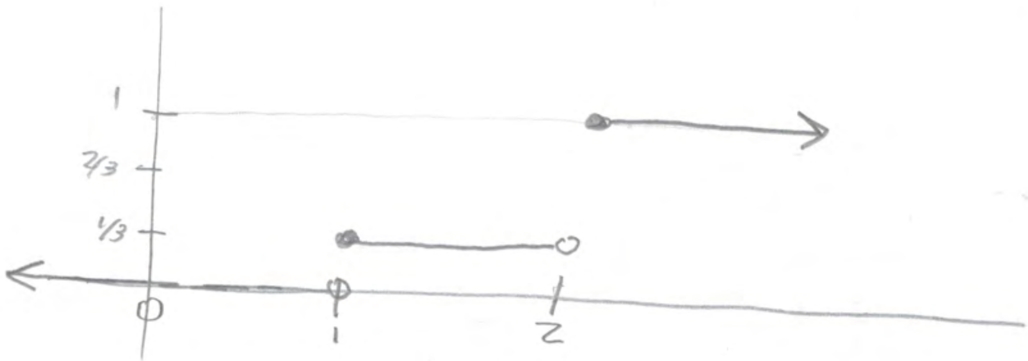
   (a) 0.0000
   (b) 0.0228
   (c) 0.0540
   (d) 0.9772

Continued on page 3

6. (10 points) Let the discrete random variable X have probability function

$$p_X(x) = \begin{cases} \frac{x}{3} & \text{for } x = 1, 2 \\ 0 & \text{Otherwise} \end{cases}.$$

Graph $F_X(X)$.



Continued on page 4

7. (15 points) Let the continuous random variables X and Y have joint cumulative distribution function

$$F_{X,Y}(x,y) = \begin{cases} 1 - e^{-2x} - e^{-y} + e^{-(2x+y)} & \text{for } x \geq 0 \text{ and } y \geq 0 \\ 0 & \text{Otherwise} \end{cases}.$$

Find the joint probability density function $f_{X,Y}(x,y)$. Don't forget to sprcify where it is non-zero.

For $x \geq 0, y \geq 0$

$$\frac{\partial^2 F}{\partial x \, \partial y} = \frac{\partial}{\partial x}\left(0 + 0 - e^{-y}(-1) + e^{-(2x+y)}(-1)\right)$$

$$= \frac{\partial}{\partial x}\left(e^{-y} - e^{-(2x+y)}\right)$$

$$= 0 - e^{-(2x+y)}(-2)$$, so

$$f_{XY}(x,y) = \begin{cases} 2e^{-(2x+y)} & \text{for } x \geq 0, \ y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

8. (15 points) Let X be a continuous random variable with density $F_X(x)$ and cumulative distribution function $F_X(x)$. Prove that $\lim_{x\to\infty} F_X(x) = 1$.

Because X is specifically a continuous random variable, you are proving a special case of something on the formula sheet — so you can't use the general fact on the formula sheet. This problem is a lot easier if you use the fact that X is continuous.

$$\lim_{x\to\infty} F_X(x) = \lim_{x\to\infty} \int_{-\infty}^{x} f_X(t)\,dt$$

$$\int_{-\infty}^{\infty} f_X(t)\,dt = 1$$

Continued on page 6

9. Let $X \sim \text{Gamma}(\alpha, \lambda)$, and let $Y = X/9$.

   (a) (12 points) Find $f_Y(y)$, the probability density of Y. Show your work. Do not forget to specify where the density is non-zero.

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} P(Y \le y) = \frac{d}{dy} P(X/9 \le y)$$

$$= \frac{d}{dy} P(X \le 9y) = \frac{d}{dy} F_X(9y)$$

$$= f_X(9y) \cdot 9 = \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda 9 y} (9y)^{\alpha-1} \cdot 9 \text{, so}$$

This is good

$$f_Y(y) = \begin{cases} \frac{(9\lambda)^\alpha}{\Gamma(\alpha)} e^{-(9\lambda)y} y^{\alpha-1} & \text{for } y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

   (b) (3 points) Y has a Gamma distribution with (fill in the blanks)

$\alpha_y = \alpha$ and $\lambda_y = 9\lambda$

Continued on page 7

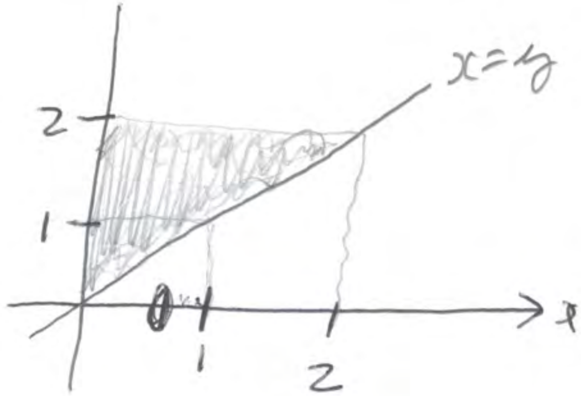
10. (20 points) Let the continuous random variables X and Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{xy}{2} & \text{for } 0 \le x \le y \le 2 \\ 0 & \text{Otherwise} \end{cases}$$

Find the marginal density $f_X(x)$. Show your work. Be sure to specify where the density is non-zero.

For $0 \le x \le 2$

$$f_X(x) = \int_x^2 \frac{xy}{2}\, dy$$



$$= \frac{x}{2}\int_x^2 y\, dy$$

$$= \frac{x}{2}\,\frac{y^2}{2}\Big|_x^2 = \frac{x}{4}(4 - x^2), \text{ so}$$

$$f_X(x) = \begin{cases} x - \frac{x^3}{4} & \text{for } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$