Name ______________________________

Student Number ______________________________

STA 256 f2019 LEC0101 Test 2

Tutorial Section (Circle One)

TUT0101 Mon. 3-4 DH 2080 Ali	TUT0102 Mon. 4-5 DH 2080 Dashvin	TUT0103 Mon. 5-6 IB 360 Dashvin	TUT0104 Mon. 6-7 IB 240 Ali	TUT0105 Wed. 4-5 IB360 Marie
TUT0106 Wed. 5-6 IB 360 Marie	TUT0107 Fri. 9-10 IB 200 Crendall	TUT0108 Fri. 10-11 DH 2070 Ali	TUT0109 Fri. 10-11 DV 3093 Cendall	TUT0110 Fri. 11-12 DH 2070 Ali
TUT0111 Fri. 11-12 DV 3093 Crendall	TUT0112 Fri. 12-1 DV 2070 Crendall	TUT0113 Fri. 4-5 DV 3093 Karan	TUT0114 Fri. 5-6 IB 360 Karan	TUT0115 Fri. 6-7 IB 360 Karan
TUT0116 Wed. 11-12 DH 2070 Ana	TUT0117 Wed. 12-1 IB 260 Ana			

Question	Value	Score
1-5		
6		
7		
8		
9		
10		
Total = 100 Points		

Circle the alternative that is *closest* to the correct answer.

1. (5 points) Let X have a normal distribution with parameters $\mu = 100$ and $\sigma^2 = 400$. What is $P(60 < X < 140)$? Circle the letter.

   (a) 0.0228
   (b) 0.0456
   (c) 0.9544
   (d) 0.9772

2. (5 points) Let X have a continuous Uniform(0,1) distribution. What is $P(-1/4 < X < 1/4)$? Circle the letter.

   (a) 0.00
   (b) 0.25
   (c) 0.50
   (d) 0.75

3. (5 points) Roll two fair dice, one red and one green. Let X be the number showing on the red die and Y be the number showing on the green die. What is $F_{X,Y}(8, 2)$? Circle the letter.

   (a) 0.000
   (b) 0.333
   (c) 0.667
   (d) 1.000

4. (5 points) Let X have a Geometric distribution with parameter $\theta = 1/4$. What is $F_X(5)$? Circle the letter.

   (a) 0.1780
   (b) 0.2373
   (c) 0.7627
   (d) 0.8220

5. (5 points) Let Z have a Standard Normal distribution; that is, $\mu = 0$ and $\sigma^2 = 1$. What is $P(Z = -2)$? Circle the letter.

   (a) 0.0000
   (b) 0.0228
   (c) 0.0540
   (d) 0.9772

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6. (10 points) Let the discrete random variable X have probability function

$$p_X(x) = \begin{cases} \frac{x}{3} & \text{for } x = 1, 2 \\ 0 & \text{Otherwise} \end{cases}.$$

Graph $F_X(X)$.

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7. (15 points) Let the continuous random variables X and Y have joint cumulative distribution function

$$F_{X,Y}(x,y) = \begin{cases} 1 - e^{-2x} - e^{-y} + e^{-(2x+y)} & \text{for } x \geq 0 \text{ and } y \geq 0 \\ 0 & \text{Otherwise} \end{cases}.$$

Find the joint probability density function $f_{X,Y}(x,y)$. Don't forget to sprcify where it is non-zero.

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8. (15 points) Let X be a continuous random variable with density $F_X(x)$ and cumulative distribution function $F_X(x)$. Prove that $\lim_{x \to \infty} F_X(x) = 1$.

   Because X is specifically a continuous random variable, you are proving a special case of something on the formula sheet — so you can't use the general fact on the formula sheet. This problem is a lot easier if you use the fact that X is continuous.

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9. Let $X \sim \text{Gamma}(\alpha, \lambda)$, and let $Y = X/9$.

   (a) (12 points) Find $f_Y(y)$, the probability density of Y. Show your work. Do not forget to specify where the density is non-zero.

   (b) (3 points) Y has a Gamma distribution with (fill in the blanks)

   $\alpha_y =$ and $\lambda_y =$

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10. (20 points) Let the continuous random variables X and Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{xy}{2} & \text{for } 0 \leq x \leq y \leq 2 \\ 0 & \text{Otherwise} \end{cases}$$

Find the marginal density $f_X(x)$. Show your work. Be sure to specify where the density is non-zero.

Total Marks = 100 points