Name ________________________________

Student Number ________________________________

Tutorial Section ________________________________

STA 256 f2018 Test 3

Question	Value	Score
1	15	
2	25	
3	30	
4	15	
5	15	
Total = 100 Points		

15 points

1. Let X and Y be discrete random variables, so that to calculate expected values, you use summation. Show that if X and Y are independent, then $Cov(X, Y) = 0$. Be very clear about where you use independence. *In your answer, draw an arrow to the place where you use independence, and write "This is where I use independence."* If you need something that is not on the formula sheet, prove it.

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25 points

2. Let X_1 and X_2 be independent continuous random variables with densities

$$f_{x_1}(x_1) = \begin{cases} e^{-x_1} & \text{for } x_1 \geq 0 \\ 0 & \text{for } x_1 < 0 \end{cases} \qquad \text{and} \qquad f_{x_2}(x_2) = \begin{cases} e^{-x_2} & \text{for } x_2 \geq 0 \\ 0 & \text{for } x_2 < 0 \end{cases}$$

(a) Find the joint density of $Y_1 = \frac{X_1}{X_2}$ and $Y_2 = X_2$. Do not forget to indicate where the joint density is non-zero.

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(b) Find the density of $Y_1 = \frac{X_1}{X_2}$. Do not forget to indicate where the density of Y_1 is non-zero.

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30 points

3. Let $X_1, \ldots, X_n$ be independent random variables with expected value μ and variance σ^2. Their distribution is unspecified, but it might not be normal. Let $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, the sample mean.

   (a) Find $E(\overline{X})$. Show your work. Circle your answer.

   (b) Find $Var(\overline{X})$. Show your work. Circle your answer.

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(c) Suppose $X_1, \ldots, X_n$ are normally distributed. Use moment-generating functions to find the distribution of $\overline{X}$, including the parameters. **Show your work.** At the end of your answer, write "$\overline{X} \sim$" ... and write the distribution.

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15 points

4. The discrete random variables X and Y have joint distribution

	$x=1$	$x=2$	$x=3$	$x=4$	$x=5$
$y=1$	1/14	1/14	1/14	1/14	1/14
$y=2$	0	1/14	1/14	1/14	1/14
$y=3$	0	0	1/14	1/14	1/14
$y=4$	0	0	0	1/14	1/14

(a) What is $E(Y)$? The answer is a number. Show your work.

(b) What is $E(Y|X=3)$? The answer is a number.

(c) What is $E(X|Y=3)$? The answer is a number.

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15 points 5. Let X have a Poisson distribution with parameter $\lambda > 0$.

(a) For what values of λ does $E(X!)$ exist?

(b) For values of λ satisfying the condition you gave above, what is $E(X!)$? The answer is a formula involving λ. Show your work. Circle your answer.

Total Marks = 100 points