Name Jerry

Student Number

Tutorial Section

STA 256 f2018 Test 2

Question	Value	Score
1	10	
2	10	
3	25	
4	15	
5	20	
6	20	
Total = 100 Points		

10 points 1. Let X have a Poisson distribution with $\lambda = 2$. What is $F_x(1.34)$? The answer is a number. Show some work. **Circle your answer**.

$$F(1.34) = P(X \le 1.34) = P(X=0) + P(X=1)$$

$$= e^{-2}\frac{2^0}{0!} + e^{-2}\frac{2^1}{1!} = e^{-2}(1+2)$$

$$= 3e^{-2} = 0.406$$

10 points 2. Prove that the Binomial probabilities sum to one.

$$(p + 1-p)^n = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$

$$\parallel$$

$$1$$

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25 points

3. The continuous random variable X has cumulative distribution function

$$F_x(x) = \begin{cases} 1 - \frac{1}{x^3} & \text{for } x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is $P(-1 < X < 2)$? The answer is a number. **Circle your answer.**

$$P(-1 < X < 2) = F(2) = 1 - \frac{1}{2^3} = 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$

(b) Find the probability density function $f_x(x)$. Show a little work. Do not forget to indicate where the density is non-zero.

For $x \geq 1$, $f_x(x) = \frac{d}{dx} F(x) = \frac{d}{dx}\left(1 - x^{-3}\right)$

$= (-1)(-3)x^{-4} = \frac{3}{x^4}$, so

$$f_x(x) = \begin{cases} \frac{3}{x^4} & \text{for } x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

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15 points

4. Let X be a normally distributed random variable with $\mu = 100$ and $\sigma = 15$. What is $P(100 < X \leq 120)$? The answer is a number. Show your work. **Circle your answer**.

$$P(100 < X < 120)$$

$$= P\left(\frac{100-100}{15} < \frac{X-\mu}{\sigma} < \frac{120-100}{15}\right)$$

$$= P\left(0 < Z < \frac{20}{15}\right) = P(0 < Z < 1.33)$$

where $Z \sim N(0,1)$

$$= F_Z(1.33) - F_Z(0) = 0.9082 - 0.5$$

$$= 0.4082$$

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20 points

5. The random variable X has probability density function $f_x(x) = \frac{e^x}{(1+e^x)^2}$, for all real x. What is the cumulative distribution function $F_x(x)$? Show your work.

$$F_X(x) = \int_{-\infty}^{x} \frac{e^t}{(1+e^t)^2}\,dt$$

$u = 1+e^t \quad du = e^t dt$

t	u
x	$1+e^x$
$-\infty$	1

$$= \int_1^{1+e^x} u^{-2}\,du$$

$$= \left.\frac{u^{-1}}{-1}\right|_1^{1+e^x} = (-1)\left(\frac{1}{1+e^x} - \frac{1}{1}\right)$$

$$= 1 - \frac{1}{1+e^x}$$

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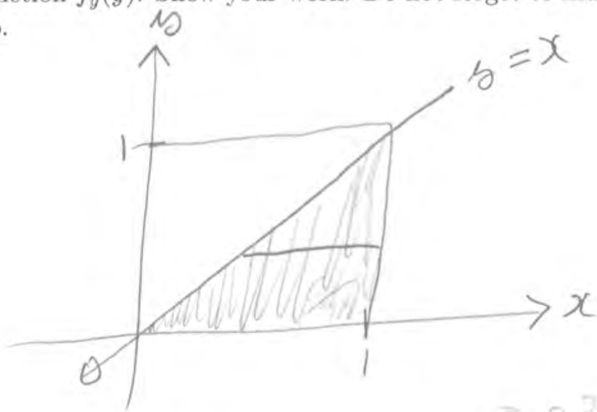
20 points

6. The continuous random variables X and Y have joint probability density function

$$f_{xy}(x,y) = \begin{cases} 10\,x^2y & \text{for } 0 \le x \le 1 \text{ and } 0 \le y \le x \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal density function $f_y(y)$. Show your work. Do not forget to indicate where the density is non-zero.



For $0 < y < 1$,

$$f_Y(y) = \int_y^1 10\,x^2 y\,dx$$

$$= 10y \int_y^1 x^2\,dx$$

$$= 10y\,\frac{x^3}{3}\Big|_y^1 = \frac{10}{3}\,y\,(1^3 - y^3) = \frac{10}{3}\,y(1-y^3),$$

So

$$f_Y(y) = \begin{cases} \frac{10}{3}\,y(1-y^3) & \text{for } 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$