Name Jenny

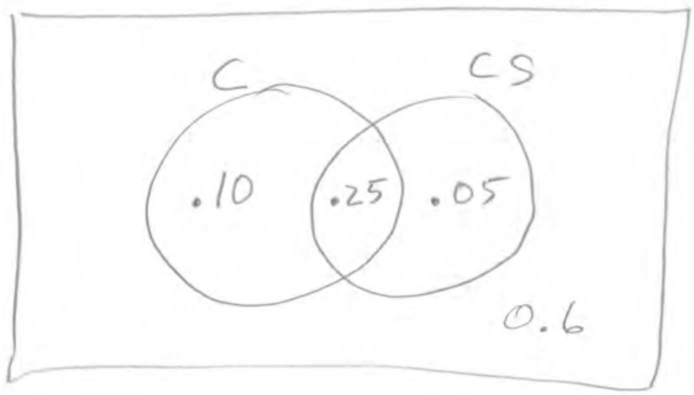
Student Number

STA 256 f2018 Test 1

Question	Value	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	20	
7	30	
Total = 100 Points		

10 points

1. In an entering university class, 35% of Science students take calculus and 30% take computer science. If 25% take both calculus and computer science and a student is chosen at random, what is the probability that she is not taking calculus and not taking computer science? The answer is a number. Show some work. **Circle your answer**.



0.6

Continued on page 3

10 points

2. Five guests at a party throw their coats and hats on the bed. After the party, they each randomly choose a coat, and randomly and independently choose a hat. What is the probability that they all get the correct coat and hat? Just write down the answer. There is no need to show any work. Do not bother with your calculator.

$$\frac{1}{5!\,5!}$$

10 points

3. A jar contains 10 black balls and 5 white balls. If four balls are randomly selected *without replacement*, what is the probability of at least one white ball? Just write down the answer. You do not need to simplify.

$$1 - \frac{\binom{10}{4}}{\binom{15}{4}}$$

10 points

4. A coin with $P(\text{Head}) = p$ is tossed 20 times. What is the probability that exactly 9 heads are observed? Just write down the answer. You do not need to simplify.

$$\binom{20}{9} p^9 (1-p)^{11}$$

10 points

5. A fair die is rolled until the first 2 appears. What is the probability that this happens on the third roll? The answer is a number. Show a little bit of work. **Circle your answer**.

$$\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} = \frac{25}{216} = 0.1157$$

Continued on page 4

20 points

6. A jar contains five blue balls and three red balls. One ball is randomly selected. It is put back into the jar, and another ball of the same colour is also placed in the jar. The balls are thoroughly mixed, and then a second ball is selected at random. If the second ball is red, what is the probability that the first ball was blue? The answer is a number. Show your work. **Circle your answer**.

$$P(B_1 \mid R_2) = \frac{P(B_1 \cap R_2)}{P(R_2)}$$

OR go straight to Bayes' Theorem

$$= \frac{P(R_2 \mid B_1)P(B_1)}{P(R_2 \mid B_1)P(B_1) + P(R_2 \mid R_1)P(R_1)}$$

$$= \frac{\frac{3}{9}\cdot\frac{5}{8}}{\frac{3}{9}\cdot\frac{5}{8} + \frac{4}{9}\cdot\frac{3}{8}} = \frac{15}{15+12}$$

$$= \frac{15}{27} = \frac{5}{9} = 0.\overline{55}$$

Continued on page 5

30 points

7. In this question, you are asked to prove the Law of Total Probability. You may use anything from the formula sheet *except* the Law of Total Probability. Let $\Omega = \cup_{k=1}^{\infty} B_k$, where $B_i \cap B_j = \emptyset$ for $i \neq j$, and $P(B_k) > 0$ for all k. Using the formula sheet and the *tabular format illustrated in lecture*, prove $P(A) = \sum_{k=1}^{\infty} P(A|B_k)P(B_k)$.

step	Justification
$P(A) = P(A \cap \Omega)$	$A = A \cap \Omega$
$= P(A \cap \bigcup_{j=1}^{\infty} B_j)$	Substitution
$= P(\bigcup_{j=1}^{\infty} (A \cap B_j))$	Distributive Law from formula sheet
Since the $A \cap B_j$ are disjoint	Because the B_j are disjoint
$= \sum_{j=1}^{\infty} P(A \cap B_j)$	Axiom 3
$= \sum_{j=1}^{\infty} P(A \mid B_j) P(B_j)$	$P(A \cap B) = P(A \mid B) P(B)$ multiplication Law from formula sheet

Total Marks = 100 points