Sets[1]

STA 256: Fall 2018

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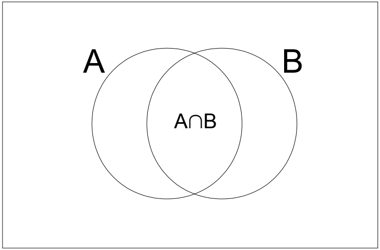
Statistical Experiment

A statistical experiment is a procedure whose outcome is not known in advance with certainty.

Sample Space: set of outcomes $\omega \in \Omega$

- Sell 500 lottery tickets, pick the winning number.
  $\Omega = \{1, 2, \ldots, 500\}$
- Hold your breath as long as you can.
  $\Omega = \{t : t \geq 0\}$
- Pick coin or die from jar, roll or toss.
  $\Omega = \{H, T, 1, 2, 3, 4, 5, 6\}$

Event: Set of outcomes, $A \subset \Omega$



- $A \cap B = \{\omega \in \Omega : \omega \in A \text{ and } \omega \in B\}$
- A and B are said to be *disjoint* if $A \cap B = \emptyset$
- $A \cup B = \{\omega \in \Omega : \omega \in A \text{ or } \omega \in B\}$
- $A^c = \{\omega \in \Omega : \omega \notin A\}$

Set Laws

No proofs, just Venn diagrams at most

- Commutative: $A \cup B = B \cup A$, $A \cap B = B \cap A$
- Associative
  - $(A \cup B) \cup C = A \cup (B \cup C)$,
  - $(A \cap B) \cap C = A \cap (B \cap C)$
- Distributive (like multiplication)
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

De Morgan Laws

Not in the text

- $(A \cap B)^c = A^c \cup B^c$
- $(A \cup B)^c = A^c \cap B^c$
- Rule: complement and flip $\cup\cap$

Extend the notation to larger number of sets

Not in the text

Distributive laws

- $A \cap \left(\cup_{j=1}^{n} B_j \right) = \cup_{j=1}^{n} (A \cap B_j)$, or even
- $A \cap \left(\cup_{j=1}^{\infty} B_j \right) = \cup_{j=1}^{\infty} (A \cap B_j)$

  and
- $A \cup \left(\cap_{j=1}^{n} B_j \right) = \cap_{j=1}^{n} (A \cup B_j)$
- $A \cup \left(\cap_{j=1}^{\infty} B_j \right) = \cap_{j=1}^{\infty} (A \cup B_j)$

De Morgan Laws (complement and flip)

- $(\cap_{j=1}^{\infty} A_j)^c = \cup_{j=1}^{\infty} A_j^c$
- $(\cup_{j=1}^{\infty} A_j)^c = \cap_{j=1}^{\infty} A_j^c$

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