Sample Questions: Joint Distributions Part Two

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1. Let X and Y be continuous random variables. Prove that X and Y are independent if and only if $f_{xy}(x,y) = f_x(x)\, f_y(y)$.

2. Let X and Y be discrete random variables. Prove that if $p_{xy}(x, y) = p_x(x)\, p_y(y)$, then X and Y are independent.

3. Let X and Y be discrete random variables. Prove that if X and Y are independent, then $p_{xy}(x, y) = p_x(x)\, p_y(y)$.

4. Let $p_{xy}(x, y) = \frac{|x-2y|}{19}$ for $x = 1, 2, 3$ and $y = 1, 2, 3$, and zero otherwise.

(a) What is $p_{y|x}(1|2)$?

(b) What is $p_{x|y}(1|2)$?

(c) Are x and y independent? Answer Yes or No and prove your answer.

5. Let $f_{x,y}(x,y) = \begin{cases} 2e^{-(x+y)} & \text{for } 0 \leq x \leq y \text{ and } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$

(a) Find $f_{x|y}(x|y)$.

(b) Are X and Y independent? Answer Yes or No and prove your answer.

6. Let $X \sim \text{Poisson}(\lambda_1)$ and $Y \sim \text{Poisson}(\lambda_2)$ be independent. Using the convolution formula, find the probability mass function of $Z = X + Y$ and identify it by name.

7. Let $X \sim \text{Binomial}(n_1, p)$ and $Y \sim \text{Binomial}(n_2, p)$ be independent. Using the convolution formula, find the probability mass function of $Z = X + Y$ and identify it by name.

8. Let X and Y be independent exponential random variables with parameter λ. Using the convolution formula, find the probability density function of $Z = X + Y$ and identify it by name.

9. Let X_1 and X_2 be independent standard normal random variables. Find the probability density function of $Y_1 = X_1/X_2$.

10. Use the Jacobian method to prove the convolution formula for continuous random variables.

11. Show that the normal probability density function integrates to one.

12. Prove $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

13. Let $X_1, \ldots, X_n$ be independent random variables with probability density function $f(x)$ and cumulative distribution function $F(x)$. Let $Y = \max(X_1, \ldots, X_n)$. Find the density $f_y(y)$.

14. Let $X_1, \ldots, X_n$ be independent random variables with probability density function $f(x) = e^{-x}$ for $x \geq 0$. Let $Y = \max(X_1, \ldots, X_n)$. Find the density $f_y(y)$.

15. Let $X_1, \ldots, X_n$ be independent random variables with probability density function $f(x)$ and cumulative distribution function $F(x)$. Let $Y = \min(X_1, \ldots, X_n)$. Find the density $f_y(y)$.