Sample Questions: Foundations of Probability

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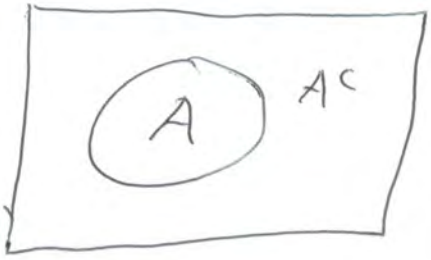
1. Prove $P(A^c) = 1 - P(A)$. Use the axioms of probability and the tabular format illustrated in lecture.

Step	Justification
$\Omega = A \cup A^c$, disjoint	
$P(\Omega) = P(A) + P(A^c)$	Axiom 3
$\Rightarrow 1 = P(A) + P(A^c)$	Axiom 1
$\Rightarrow P(A) = 1 - P(A^c)$	Math

2. Prove $P(\emptyset) = 0$. Use the axioms of probability and the tabular format illustrated in lecture.

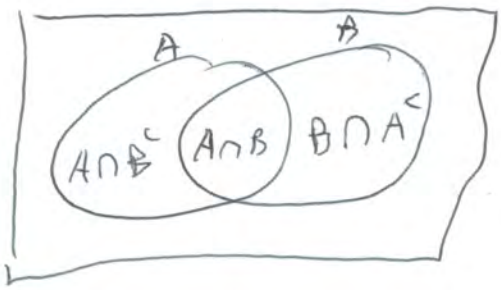
Step	Justification
$\emptyset = \Omega^c$	Set logic + democracy
$P(\emptyset) = P(\Omega^c)$ $= 1 - P(\Omega)$	Property A (Q1)
$= 1 - 1$	AXIOM 1
$= 0$	Math

3. Prove that if $A \subseteq B$ then $P(A) \leq P(B)$. Use the axioms of probability and the tabular format illustrated in lecture.

	Steps	Justification
1.	$B = A \cup (B \cap A^c)$ disjoint	
2.	$P(B) = P(A) + P(B \cap A^c)$	Axiom 3
3.	$\Rightarrow P(B \cap A^c) = P(B) - P(A)$	Math
	$P(B \cap A^c) \geq 0$	Axiom 2
	$\Rightarrow P(B) - P(A) \geq 0$	Step 3
	$P(B) \geq P(A)$	Math

4. Prove the Addition Law: $P(A \cup B) + P(A) + P(B) - P(A \cap B)$. Use the axioms of probability and the tabular format illustrated in lecture.

$A \cup B = (A \cap B^c) \cup (A \cap B) \cup (B \cap A^c)$
disjoint



1	$A = (A \cap B^c) \cup (A \cap B)$, disjoint	//
2	$B = (A \cap B) \cup (B \cap A^c)$, disjoint	//
3	$P(A \cup B) = P(A \cap B^c) + P(A \cap B) + P(B \cap A^c) + P(A \cap B) - P(A \cap B)$	Axiom 3, math
	$= P(A) + P(B) - P(A \cap B)$	Steps 1 & 2, substitution

5. If 23 out of 25 are employed, what is the probability of randomly choosing an unemployed person? The answer is a number. Circle your answer.

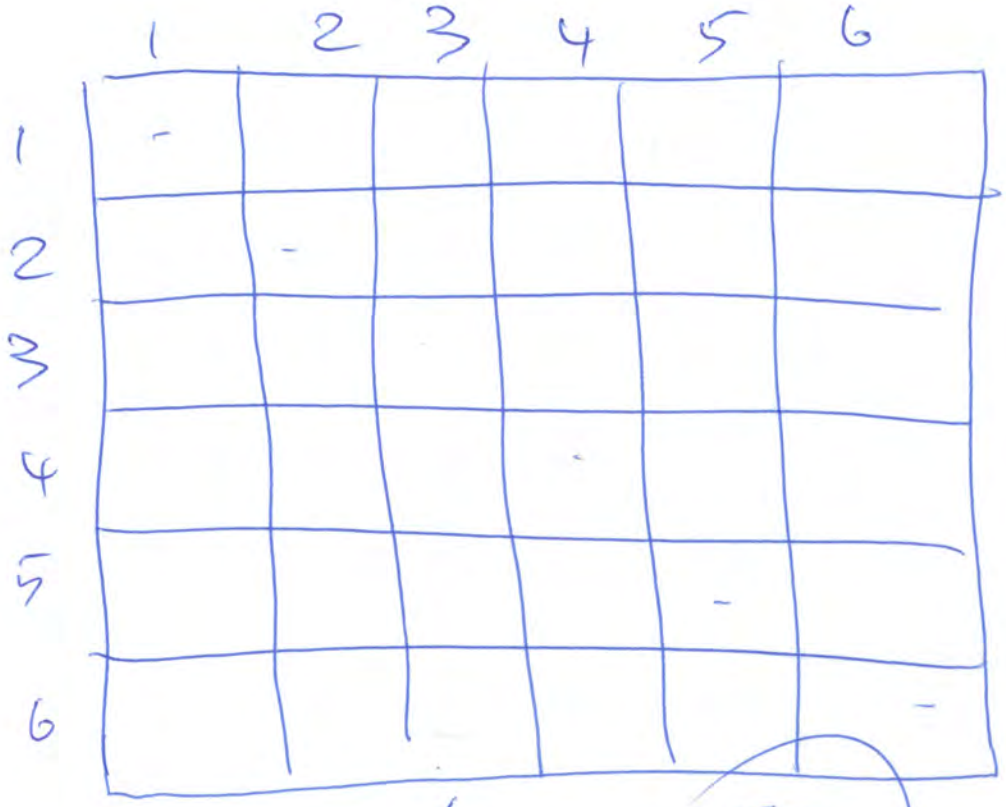
$\frac{2}{25}$ $= 1 - P(\text{Employed})$

$= 1 - \frac{23}{25}$

6. If you roll two fair dice, what is the probability of getting a sum greater than 2? The answer is a number. Circle your answer.

$$1 - \frac{1}{36} = \frac{35}{36}$$

7. If you roll two fair dice, what is the probability of getting two different numbers? Your answer is a number. Circle your answer.



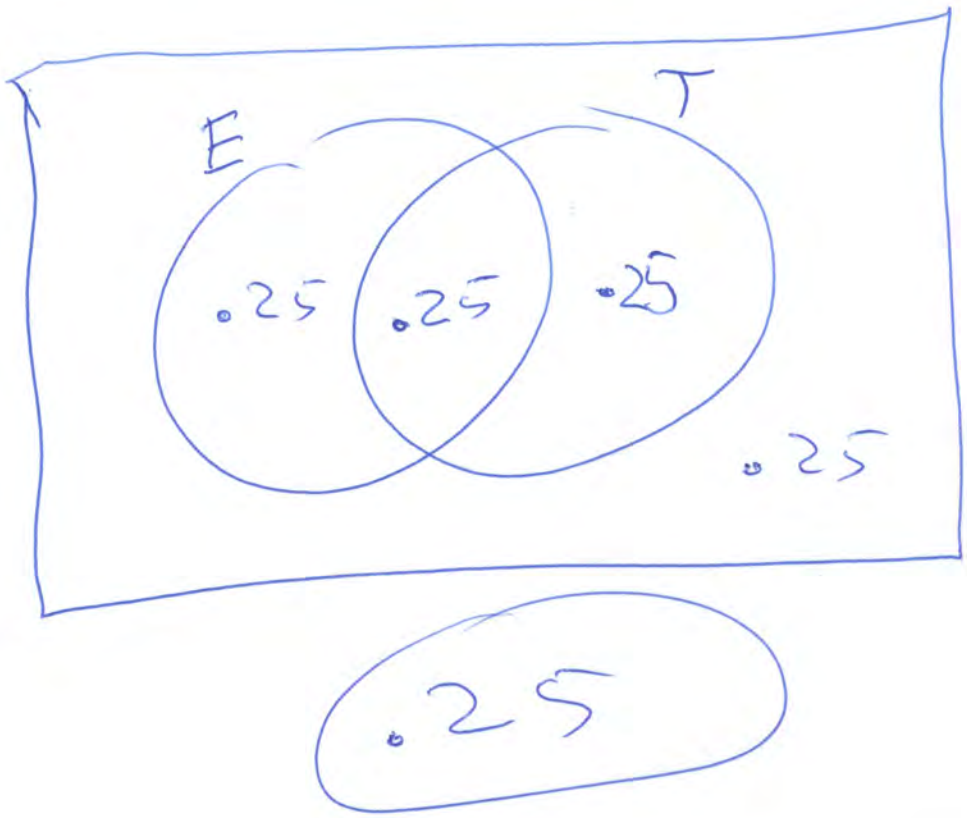
$$1 - \frac{6}{36} = \frac{5}{6}$$

8. $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$. What is $P(A \cup B)$? The answer is a number. Circle your answer.

$$P(A \cup B) = P(A) + P(B) - P(A \wedge B)$$

$$= .4 + .5 - .3 = .6$$

9. Of the cars in a used car lot, 50% have engine trouble and 50% have transmission trouble. If 25% have both problems and you buy a car at random, what is the probability that both the engine and transmission are okay? The answer is a number. Circle your answer.



10. Of the prisoners in a jail, 75% are convited murderers and 50% have been convicted of both murder and armed robbery. Twenty percent are in jail for offences other than murder or armed robbery. If you pick a prisoner at random, what is the probability that she is an armed robber?