STA 256f18 Assignment Eight[1]

Please read Sections 4.1 and 4.2 in the text, except skip 4.2.1 on measurement error.

The following homework problems are not to be handed in. They are preparation for Term Test 3 and the final exam. All textbook problems are from Chapter Four. Use the formula sheet to do the problems. On tests and the final exam, you may use anything on the formula sheet unless you are being directly asked to prove it.

1. Let X have a Bernoulli distribution, meaning $P(X = 1) = p$ and $P(X = 0) = 1 - p$. Calculate $E(X)$ and $Var(X)$.
2. Show that if $P(X \geq 0) = 1$, then $E(X) \geq 0$. Treat the discrete and continuous cases separately.
3. Do Problem 5 in the text.
4. Do Problem 1 in the text.
5. Let X and Y be independent discrete random variables. Show $E[g(X)h(Y)] = E[g(X)]E[h(Y)]$. Clearly specify where you use independence.
6. Let X and Y be discrete random variables, not necessarily independent. Show $E(aX + bY) = aE(X)+bE(Y)$. You are proving a special case of $E\left(\sum_{i=1}^n a_iX_i\right) = \sum_{i=1}^n a_iE(X_i)$ from the formula sheet, so you can't use that.
7. Show $Var(X) = E(X^2) - [E(X)]^2$.
8. Prove $Var(a + X) = Var(X)$.
9. Prove $Var(bX) = b^2Var(X)$
10. Let X have a continuous uniform distribution on (a, b). Calculate $E(X)$ and $Var(X)$.
11. Let $X \sim$ Poisson(λ). Calculate $E(X)$ and $Var(X)$. For the variance, it helps to start with $E[X(X - 1)]$.
12. Let X have a binomial distribution with parameters n and p. Calculate $E(X)$.
13. Let X have a Gamma distribution with parameters α and λ. Calculate $E(X)$ and $Var(X)$.
14. Let $X \sim N(\mu, \sigma)$. Calculate $E(X)$.
15. Do Problem 8 in the text.

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16. Do Problem 12 in the text. Show $E(X-\xi)=0$. Symmetry means $f(\xi-x)=f(\xi+x)$. Split the integral at ξ and change variables.

17. Do Problem 13 in the text. This is a double integral. Sketch the region of integration and switch order of integration.

18. Do Problem 16 in the text.

19. Do Problem 21 in the text.

20. Do Problem 31 in the text.

21. Markov's inequality says that if $P(Y \geq 0)=1$, then $P(Y \geq t) \leq E(Y)/t$. Prove it for the case where Y is a discrete random variable.

22. Chebyshev's inequality says if X is a random variable with $E(X)=\mu$ and $Var(X)=\sigma^2$, then for any $k>0$, $P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$. Use Markov's inequality to prove it.
