May 2013

Ph.D. COMPREHENSIVE EXAMINATIONS
DEPARTMENT OF STATISTICS
UNIVERSITY OF TORONTO

APPLIED STATISTICS COMPREHENSIVE EXAMINATION

1. ***Attempt all questions*** (total # of questions = 6).
   It is not necessary to completely solve every problem to achieve a good performance. *Emphasize what you do know.*
2. Please work neatly and legibly.
3. ***Start each question in a new book, with your name and the number of the question on the front cover.*** If there is more than one book for a question, then also indicate which is the first book and which second, e.g., Jane Smith, Question 5, Book 1 of 2.
4. The questions are not in any special order, nor are they all of equal difficulty.
5. The problems may be improperly phrased or may contain a misprint. Should this happen, reflect it in your discussion. Faculty members are *not* available to answer questions during the exam.
6. You are NOT permitted any aids (e.g., books, notes, etc.) aside from a single non-programmable calculator.
7. Probability tables that may be useful are appended at the end of the exam paper, after all questions.
8. Good luck!

1. Rats like to eat peanuts, and it is very difficult to keep them out of storage areas — but companies must try. Federal regulations specify that the number of rat hairs in a 500g jar of peanut butter be at most 8 on average, and that a one-sided hypothesis test with the $\alpha = 0.05$ significance level be used to determine whether the standard is exceeded. So peanut butter jars are randomly sampled, and technicians carefully count the hairs using a magnifying glass. Assuming that rat hairs occur in batches of peanut butter according to a spatial Poisson process (very reasonable), the number of hairs in a jar should have a Poisson distribution with parameter λ.

   (a) Write down the formula for a convenient test statistic, and state its approximate distribution under the null hypothesis. You don't have to prove anything.

   (b) Suppose that the true mean number of rat hairs per jar is nine. If $n = 50$ jars are sampled, what is the probability of rejecting the null hypothesis? The answer is a number. Show your work. **Circle your final answer.**

   (c) Suppose again that the true mean number of rat hairs per jar is nine. What is the minimum number of peanut butter jars that must be examined in order to detect this situation by rejecting the null hypothesis with probability of at least 0.90? The answer is a number. Show your work. **Circle your final answer.**

2. Assume that in a clinical trial for an anti-arrhythmic drug, patients are randomized into a Treatment and a Control group. Patients in the treatment group receive the anti-arrhythmic drug, while those in the control group receive an inactive placebo. Cardiologists obtain an ECG for each patient for a certain time period, then count the number of extrasystolic (abnormal) heart beats that have been recorded during the specified time period. The goal is to determine whether the drug under consideration is effective in the sense of lowering the occurrence rate of abnormal events compared to those in the control group. Besides the treatment information (yes or no), age, gender and length of the time period are available as covariates for 24 patients. It is known that the average occurrences of extrasystolic heart beats is roughly proportional to the length of the time period, given other covariates the same. In a first approximation, assume the number of extrasystolic heart beats is Poisson distributed. However, it is found that the Poisson parameter is in fact also random due to individual variation.

   (a) Denote the individual Poisson parameter by Z_i. To have a constant over-dispersion, what assumptions are needed for Z_i? Calculate the dispersion for the data, as compared to a regular Poisson model.

   (b) Suppose that only main effects are significant, write down the GLM model with the canonical link and the categorical predictors expressed by dummy variables.

   (c) Suppose that the exact age of a patient is not available, but is recorded as young or old. The experimental design is balanced with respect to treatment, gender and age. Denote the saturated model up to three-way interactions as the full model and the main-effect model as the reduced model. The deviances of the full and reduced models are 32 and 48, respectively. The average number of extrasystolic heart beats under each condition is given below. Describe how you will test whether the main-effect only model is adequate by specifying the value of the test statistic and its distribution under the null hypothesis.

(Male)	Treatment	Control	(Female)	Treatment	Control
Young	6.2	5.8	Young	3.8	4.3
Old	10.2	8.3	Old	0.0	6.1

3. The Ontario government selects a random sample of q grade schools. From each school, a random sample of k students is selected and given a reading test. School is the explanatory variable. Because the values of this variable represent a random sample from a larger populations, a *random effects* model is appropriate. A standard version that applies to this situation is

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij},$$

where

μ is an unknown constant parameter.

$\tau_i \sim N(0, \sigma_\tau^2)$ and $\epsilon_{ij} \sim N(0, \sigma^2)$.

τ_i and ϵ_{ij} are all independent, $i = 1, \ldots q$ and $j = 1, \ldots, k$.

(a) What is the distribution of Y_{ij}? Just write down the answer. You need not show any work.

(b) Are the Y_{ij} all independent? Consider two cases.

(c) What is the distribution of $\overline{Y}_i = \frac{1}{k}\sum_{j=1}^{k} Y_{ij}$? State your answer; the only work you need to show is your calculation of the variance.

(d) Find $Cov(\overline{Y}_i, Y_{ij} - \overline{Y}_i)$. Show your work.

(e) Define $SSTR = k\sum_{i=1}^{q}(\overline{Y}_i - \overline{Y}_.)^2$, where $\overline{Y}_. = \frac{1}{q}\sum_{i=1}^{q}\overline{Y}_i$. Find the distribution of $\frac{SSTR}{\sigma^2+k\sigma_\tau^2}$. Hint: You can make this a very easy problem by using a well-known fact without proof.

(f) Define $SSE = \sum_{i=1}^{q}\sum_{j=1}^{k}(Y_{ij} - \overline{Y}_i)^2$. Find the distribution of $\frac{SSE}{\sigma^2}$. Again you may use a well-known fact to make the problem easier, but *do not forget your answer to 3b.*

(g) The proportion of variance in an observation Y_{ij} that is explained by School is $\frac{\sigma_\tau^2}{\sigma_\tau^2+\sigma^2}$. Give a reasonable estimator for this quantity; show some work.

(h) What null hypothesis would you use to test for the effect of school on students' reading scores? State the null hypothesis in symbolic form; that is, it's a statement in terms of Greek letters.

(i) An exact (not large-sample) test is available for this hypothesis. Give a formula for the test statistic. Also state its distribution under H_0, including the degrees of freedom. Briefly indicate why it has the distribution you claim.

(j) Show that the power of this test is based on a *central* rather than a non-central F distribution.

(k) Suppose that the government's primary interest is in testing whether School has any effect at all on average reading score. Since resources are limited, would you advise the government to spend money on sampling more schools, or more students per school? Why?

4. If x is the dose level of a toxin, subjects exposed to this toxin are observed to respond or not respond to it, leading to a binary response. If a group of m individuals is exposed to the toxin at dose x, let y denote the number of individuals that respond and assume $y \sim Bin(m, \pi)$. Suppose that the *unobservable* individual threshold T has the cumulative distribution function $F(u) = \exp(\frac{u-\mu}{\sigma})/\{1 + \exp(\frac{u-\mu}{\sigma})\}$.

   (a) Justify how to model the dose effect using a generalized linear model. Specify the model elements and express μ and σ in terms of regression coefficients.

   (b) Suppose that 20 distinct dose levels are randomized to male (coded as 1) and female (coded as 0) individuals of equal size. Consider a model without interaction, some R output from the model in (a) is given below. Fill in the missing degree of freedoms and obtain the estimate of ED50 and its 95% confidence interval for male population.

```
              Estimate      Std. Error
(Intercept)   -8.7          1.0
          x   10.0          0.7
        sex   -3.0          0.5
(Dispersion Parameter for Binomial family taken to be 1.)
Residual Deviance: 34 on ? degrees of freedom

Correlation of Coefficients:
              (Intercept)  x
        x      0.25
        sex    0.15        0.10
```

5. Denote $y = (y_1, \ldots, y_n)^\top$ as the vector of observations with the mean vector $\mu = (\mu_1, \ldots, \mu_n)^\top$, and the vector of regression coefficients $\beta = (\beta_1, \ldots, \beta_p)^\top$.

   (a) Write the estimating equation based on quasi-likelihood, and express the approximate solution $\hat{\beta}$ of this estimating equation based on Fisher-scoring algorithm.

(b) Let $\mathcal{L} = \{h(y;\beta) : h(y;\beta) = H^\top(y - \mu(\beta))\}$ be the the class of linear estimating equations, where the $n \times p$ matrix H may be a function of β but not of y. Assume the root of $h(y;\beta) = 0$ to be unique. Write down the approximate solution $\tilde{\beta}$ obtained from an estimating equation belonging to the class $\mathcal{L}$ based on Fisher scoring algorithm.

(c) Show that the estimating equation based on quasi-likelihood is optimal within the class $\mathcal{L}$ using Fisher-scoring algorithm. (*Hint: Use the fact from linear algebra: A and B both positive-definite matrices, then $A - B \geq 0$ implies $A^{-1} - B^{-1} \leq 0$, where "≥ 0" means non-negative definite.*)

6. Several years ago, the university wanted to compare marking practices on the three U of T campuses. One of the data files had just campus, high school grade point average, 4th year university grade point average, and the number of credits on which university grade point average was based. First, take a look at just the R commands, so you can see what was done.

```
########## Read the data ##########
tricampus = read.csv('TriCampus.csv') # Read data from MS Excel spreadsheet
head(tricampus)
attach(tricampus) # Now variable names are available

########## Descriptive statistics ##########
mean(HSGPA); mean(UNIVGPA)
min(HSGPA); max(HSGPA)
table(Campus)
# Mean University GPA by campus
aggregate(UNIVGPA,by=list(Campus),FUN=mean)
# Mean High School GPA by campus
aggregate(HSGPA,by=list(Campus),FUN=mean)

##########  Make dummy variables, interaction terms ##########
n = length(Campus)
c1 = c2 = c3 = numeric(n)
c1[Campus=='SG'] = 1; c2[Campus=='UTM'] = 1; c3[Campus=='UTSC'] = 1
c1HSGPA = c1 * HSGPA; c2HSGPA = c2 * HSGPA; c3HSGPA = c3 * HSGPA

##########  Fit and compare some models ##########
model1 = lm(UNIVGPA~c2+c3); summary(model1)
summary(lm(UNIVGPA~c1+c2))
model2 = lm(UNIVGPA~HSGPA); summary(model2)
model3 = lm(UNIVGPA~HSGPA+c2+c3); summary(model3)
summary(lm(UNIVGPA~HSGPA+c1+c2))
```

```
anova(model1,model3)
anova(model2,model3)
model4 = lm(UNIVGPA~HSGPA+c2+c3+c2HSGPA+c3HSGPA); summary(model4)
anova(model3,model4)
summary(lm(UNIVGPA~HSGPA+c1+c2+c1HSGPA+c2HSGPA))
```

The full R session may be found after the questions below.

(a) Consider a model (from the printout) that compares mean GPA at the three campuses, completely ignoring High School marks.

i. Write the model in statistical terms. Begin with "$Y_i = \beta_0 + \ldots$". Include the distributional assumptions. Identify any variables that appear in the data file or have been calculated from variables in the data file; for example "Y_i is ...".

ii. In terms of the parameters (Greek letters) of your model, what is the expected 4th year grade point average for a student from the UTSC campus?

iii. What is the null hypothesis you would test in order to decide whether there is a difference in mean 4th year GPA across the three campuses? Your answer is an expression involving Greek letters.

iv. Give the computed value of the test statistic; the answer is a number from the printout. Do these results provide strong evidence against H_0?

v. Copy the table below into your answer book, and write the computed values of the test statistics for the pairwise comparisons between means.

	Test Statistic	Reject H_0 at $\alpha = 0.05$ with Bonferroni correction (Yes or No)?
SG *vs.* UTM		
SG *vs.* UTSC		
UTM *vs.* UTSC		

vi. In plain, non-technical language, what do you conclude from the analysis based on this first model? Your answer is something about grade point average at the three campuses. State your answer in clear terms that the average reader of a newspaper could understand. Marks will be deducted for the use of statistical terminology like "null hypothesis", "statistically significant" and so on.

(b) Now consider a model that includes high school marks, with the slope of the regression line relating high school marks to university marks being the *same* on all three campuses. Again, this is one of the models from the printout.

i. Write the model in statistical terms. Begin with "$Y_i = \beta_0 + \ldots$". Include the distributional assumptions. Identify any variables that appear in the data file or have been calculated from variables in the data file; for example "Y_i is ...".

ii. Write three regression equations, one for each campus. Use the notation from your model immediately above.

iii. For each campus, predict the 4th year university grade point average for students with a high school average of 84%. Your answer is *three* numbers, one for each campus.

iv. What null hypothesis would you test to answer the following question: Controlling for (holding constant) high school grade point average, is there a difference in mean 4th year GPA across the three campuses? Your answer is an expression involving Greek letters.

v. Give the computed value of the test statistic for that last null hypothesis; the answer is a number from the printout. Do these results provide strong evidence against H_0?

vi. Copy the table below into your answer book, and write the computed values of the test statistics for the pairwise comparisons between means. Again, this time you're comparing means *controlling for* high school average.

	Test Statistic	Reject H_0 at $\alpha = 0.05$ with Bonferroni correction (Yes or No)?
SG *vs.* UTM		
SG *vs.* UTSC		
UTM *vs.* UTSC		

vii. In plain, non-technical language, what do you conclude from the analysis based on this model? Your answer is something about grade point average at the three campuses. State your answer in clear terms that the average reader of a newspaper could understand. Marks will be deducted for the use of statistical terminology like "null hypothesis," "statistically significant" and so on. But you may start with "Allowing for high school marks, ...".

(c) Finally, consider a model in which the slopes as well as the intercepts might be different for the three campuses. Again, this is one of the models from the computer output.

i. Based on the fitted model, give three equations for predicting university GPA from high school GPA; there should be one linear equation for each campus. The slopes and intercepts should be *numbers* that come from the computer output.

ii. Copy the table below into your answer book, and write predicted 4th year grade point averages in the table. For example, in thge upper left cell you would write the predicted GPA for a student from the SG campus, with a high school average of 65%.

Campus	High School GPA 65	95
SG		
UTM		
UTSC		

iii. Give the test statistic and p-value for testing equality of the three slopes. You are being asked for two numbers from the computer output. Is there convincing evidence that the three slopes are different?

iv. Copy the table below into your answer book, and write the computed values of the test statistics for the pairwise comparisons between *slopes* of the three regression lines.

	Test Statistic	Reject H_0 at $\alpha = 0.05$ with Bonferroni correction (Yes or No)?
SG *vs.* UTM		
SG *vs.* UTSC		
UTM *vs.* UTSC		

v. It is hard to use non-statistical language to talk about differences in slopes of regression lines. But you can turn it around and talk about whether differences between campuses in expected 4th year grade point average *depend* on high school grade point average. Please summarize the results using this language, avoiding technical statistical terms as much as possible.

The rest of this question is computer input and output.

```
> ########## Read the data ##########
> tricampus = read.csv('TriCampus.csv') # Read data from MS Excel spreadsheet
> head(tricampus)
  Campus HSGPA UNIVGPA Credits
1     SG  99.5    3.90    16.5
2     SG  99.3    3.97    20.5
3     SG  99.2    3.82    20.0
4     SG  99.0    3.99    20.0
5     SG  99.0    3.55    19.0
6     SG  98.8    3.78    20.0
> attach(tricampus) # Now variable names are available
>
> ########## Descriptive statistics ##########
> mean(HSGPA); mean(UNIVGPA)
[1] 83.73631
[1] 2.731247
```

```
> min(HSGPA); max(HSGPA)
[1] 63.7
[1] 99.5
> table(Campus)
Campus
  SG  UTM UTSC
2732 1002 1223
> # Mean University GPA by campus
> aggregate(UNIVGPA,by=list(Campus),FUN=mean)
  Group.1        x
1      SG 2.811783
2     UTM 2.626986
3    UTSC 2.636762
> # Mean High School GPA by campus
> aggregate(HSGPA,by=list(Campus),FUN=mean)
  Group.1        x
1      SG 85.89938
2     UTM 80.87026
3    UTSC 81.25249
>
> ##########  Make dummy variables, interaction terms ###########
> n = length(Campus)
> c1 = c2 = c3 = numeric(n)
> c1[Campus=='SG'] = 1; c2[Campus=='UTM'] = 1; c3[Campus=='UTSC'] = 1
> c1HSGPA = c1 * HSGPA; c2HSGPA = c2 * HSGPA; c3HSGPA = c3  * HSGPA
>
> ##########  Fit and compare some models ##########
>
> model1 = lm(UNIVGPA~c2+c3); summary(model1)
Call:
lm(formula = UNIVGPA ~ c2 + c3)
Residuals:
     Min       1Q   Median       3Q      Max
-1.55178 -0.44676 -0.00178  0.43822  1.36324
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.81178    0.01116 251.989   <2e-16 ***
c2          -0.18480    0.02154  -8.579   <2e-16 ***
c3          -0.17502    0.02007  -8.722   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
Residual standard error: 0.5832 on 4954 degrees of freedom
```

```
Multiple R-squared: 0.02292,Adjusted R-squared: 0.02253
F-statistic: 58.11 on 2 and 4954 DF,  p-value: < 2.2e-16

> summary(lm(UNIVGPA~c1+c2))
Call:
lm(formula = UNIVGPA ~ c1 + c2)
Residuals:
     Min       1Q   Median       3Q      Max
-1.55178 -0.44676 -0.00178  0.43822  1.36324
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.636762   0.016677 158.104   <2e-16 ***
c1           0.175021   0.020066   8.722   <2e-16 ***
c2          -0.009776   0.024852  -0.393    0.694
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
Residual standard error: 0.5832 on 4954 degrees of freedom
Multiple R-squared: 0.02292,Adjusted R-squared: 0.02253
F-statistic: 58.11 on 2 and 4954 DF,  p-value: < 2.2e-16
>
> model2 = lm(UNIVGPA~HSGPA); summary(model2)
Call:
lm(formula = UNIVGPA ~ HSGPA)
Residuals:
     Min       1Q   Median       3Q      Max
-1.84391 -0.32485  0.03261  0.35427  1.64350
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.98823    0.09491  -20.95   <2e-16 ***
HSGPA        0.05636    0.00113   49.86   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
Residual standard error: 0.4814 on 4955 degrees of freedom
Multiple R-squared: 0.3341,Adjusted R-squared: 0.3339
F-statistic:  2486 on 1 and 4955 DF,  p-value: < 2.2e-16
> model3 = lm(UNIVGPA~HSGPA+c2+c3); summary(model3)
Call:
lm(formula = UNIVGPA ~ HSGPA + c2 + c3)
Residuals:
     Min       1Q   Median       3Q      Max
-1.82545 -0.32382  0.02734  0.35216  1.73930
Coefficients:
```

```
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.337212   0.105637 -22.125  < 2e-16 ***
HSGPA        0.059942   0.001225  48.927  < 2e-16 ***
c2           0.116660   0.018730   6.228 5.10e-10 ***
c3           0.103524   0.017433   5.938 3.08e-09 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
Residual standard error: 0.4789 on 4953 degrees of freedom
Multiple R-squared: 0.3413,Adjusted R-squared: 0.3409
F-statistic: 855.4 on 3 and 4953 DF,  p-value: < 2.2e-16

> summary(lm(UNIVGPA~HSGPA+c1+c2))
Call:
lm(formula = UNIVGPA ~ HSGPA + c1 + c2)
Residuals:
     Min       1Q   Median       3Q      Max
-1.82545 -0.32382  0.02734  0.35216  1.73930
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.233688   0.100483 -22.230  < 2e-16 ***
HSGPA        0.059942   0.001225  48.927  < 2e-16 ***
c1          -0.103524   0.017433  -5.938 3.08e-09 ***
c2           0.013136   0.020413   0.644     0.52
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
Residual standard error: 0.4789 on 4953 degrees of freedom
Multiple R-squared: 0.3413,Adjusted R-squared: 0.3409
F-statistic: 855.4 on 3 and 4953 DF,  p-value: < 2.2e-16

> anova(model1,model3)
Analysis of Variance Table
Model 1: UNIVGPA ~ c2 + c3
Model 2: UNIVGPA ~ HSGPA + c2 + c3
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1   4954 1685.2
2   4953 1136.1  1    549.08 2393.8 < 2.2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

> anova(model2,model3)
Analysis of Variance Table
Model 1: UNIVGPA ~ HSGPA
```

```
Model 2: UNIVGPA ~ HSGPA + c2 + c3
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1   4955 1148.5
2   4953 1136.1  2    12.456 27.154 1.869e-12 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
>
> model4 = lm(UNIVGPA~HSGPA+c2+c3+c2HSGPA+c3HSGPA); summary(model4)
Call:
lm(formula = UNIVGPA ~ HSGPA + c2 + c3 + c2HSGPA + c3HSGPA)
Residuals:
     Min       1Q   Median       3Q      Max
-1.85999 -0.31903  0.03459  0.34989  1.82132
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.800770   0.146983 -19.055  < 2e-16 ***
HSGPA        0.065339   0.001708  38.259  < 2e-16 ***
c2           0.910432   0.261588   3.480 0.000505 ***
c3           1.126263   0.239508   4.702 2.64e-06 ***
c2HSGPA     -0.009480   0.003169  -2.992 0.002789 **
c3HSGPA     -0.012279   0.002882  -4.261 2.08e-05 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
Residual standard error: 0.478 on 4951 degrees of freedom
Multiple R-squared: 0.3441,Adjusted R-squared: 0.3434
F-statistic: 519.4 on 5 and 4951 DF,  p-value: < 2.2e-16

> anova(model3,model4)
Analysis of Variance Table
Model 1: UNIVGPA ~ HSGPA + c2 + c3
Model 2: UNIVGPA ~ HSGPA + c2 + c3 + c2HSGPA + c3HSGPA
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1   4953 1136.1
2   4951 1131.2  2    4.8246 10.558 2.659e-05 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
>
> summary(lm(UNIVGPA~HSGPA+c1+c2+c1HSGPA+c2HSGPA))
Call:
lm(formula = UNIVGPA ~ HSGPA + c1 + c2 + c1HSGPA + c2HSGPA)
Residuals:
     Min       1Q   Median       3Q      Max
```

```
-1.85999 -0.31903  0.03459  0.34989  1.82132
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.674507   0.189104  -8.855  < 2e-16 ***
HSGPA        0.053060   0.002321  22.858  < 2e-16 ***
c1          -1.126263   0.239508  -4.702 2.64e-06 ***
c2          -0.215831   0.287375  -0.751    0.453
c1HSGPA      0.012279   0.002882   4.261 2.08e-05 ***
c2HSGPA      0.002799   0.003537   0.791    0.429
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
Residual standard error: 0.478 on 4951 degrees of freedom
Multiple R-squared: 0.3441,Adjusted R-squared: 0.3434
F-statistic: 519.4 on 5 and 4951 DF,  p-value: < 2.2e-16
>
```

TABLE A Table entry for z is under the standard normal curve to the left of z.

TABLE A Standard normal probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998