ACT460H1F/STA2502 Stochastic Methods for Actuarial Science

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Office hours: Wednesdays 10am-noon, 2pm-4pm or by appointment.
Lecture times and location: Tuesdays 2-5pm, SS1087.

Prerequisites: STA347
Having STA347 is a must. If you have not taken ACT370 but did well in STA347 and other
math courses, and have some background in finance/financial economics, I will let you take
the course. According the FAS regulations, if you are missing the prerequisite you must
submit a waiver form to me for approval. The form can be downloaded from
Please submit a filled waiver form by Thursday Sept 24, or you will be removed from the
course on Friday Sept 25.

Recommended Reference Books
Financial Calculus: An Introduction to Derivative Pricing by Martin Baxter and
Andrew Rennie, Cambridge University Press (I strongly encourage you to purchase a copy
from Amazon)
Introductory Stochastic Analysis for Finance and Insurance by X. Sheldon Lin,
Wiley and Sons
Derivatives Markets by Robert Macdonald
Options, Futures and Other Derivatives by John Hull, Prentice Hall

Topics
I will use Financial Calculus as a main reference and cover the topics in the following order.
However, the material to be covered in the course will be substantially more than those in
Financial Calculus in terms of breadth and depth.

• Mathematical Foundations
  - the binomial distribution
  - the normal and lognormal distributions
  - expectation, volatility and Laplace transform
  - the central limit theorem and the law of large numbers
    time value of money, force of interest, money market account
  - stock models: binomial model and geometric brownian motion
  - actuarial pricing vs no-arbitrage pricing
  - perfect hedging: forward contract as an example

• Discrete Processes and Binomial Trees
- one-period binomial model/tree
- no-arbitrage pricing and risk-neutral probabilities
- random walk and binomial stock model
- multi-period recombining binomial tree and the distribution of the risky asset at any given time
- risk-neutral probability measure
- pricing options by backwards recursion
- the CRR formula, pricing options using the binomial distribution
- Excel and VBA programming for option pricing
- American option and valuation
- self-financing strategy, replicating portfolio
- complete and incomplete markets
- martingale
- construction of binomial models from market information
- valuation and hedging of variable annuity guarantees with binomial model
- moving towards continuous models

• Continuous Processes
  - (arithmetic) Brownian motion and properties
  - geometric Brownian motion model for stock prices
  - stochastic integral and properties
  - Ito process and SDEs
  - SDE expression of BM and GBM
  - Ito’s lemma and its applications
  - the product rule
  - martingale revisit
  - present value process and martingale
  - Fundamental Theorem of Asset Pricing
  - risk-neutral valuation
  - self-financing trading strategy and replicating portfolio in continuous time
  - application to the Black-Scholes model
  - The Black-Scholes PDE
  - greeks (Delta, Gamma, Rho and Vega) and sensitivity analysis
  - implied volatility
  - simulating stock price paths under the real-world and risk-neutral probability measures, and pricing option by simulation, using Excel and VBA
- dynamic hedging variable annuities using greeks

- Pricing Market and Fixed Income Securities
  - currency exchange and exchange forwards and options
  - options on stocks with dividends
  - zero-coupon/discount bonds and coupon bonds
  - zero rate, bond yield, short rate and forward rate, and their term structures
  - SDE for zero-coupon bonds, the Sharp ratio
  - risk-neutral valuation for bonds
  - the extended Vasicek/Hull-White short rate model and its Properties
  - the affine form of zero-coupon bond prices under Vasicek
  - calibration of the extended Vasicek model to zero curve
  - simulating short rate paths
  - the Cox, Ingersoll and Ross model and its applications
  - the Balck-Derman-Toy model
  - derivatives on discount and coupon-bearing bonds

Assignments, Test and Exam
I will give 5 homework assignments during the semester. One half of the problems in the assignments will be graded. Each assignment is worth 3% towards to the final mark. There will be 2 tutorials conducted by a TA that cover EXCEL and VBA programming skills for financial modelling. A 90-minutes written-answer midterm test will take place on Tuesday November 3 from 2:30 to 4:00. It will account for 34% of the final mark. Should you be forced to miss the test, you are required by faculty regulations to submit, within one week, appropriate documentation from the U of T Health Services to me or to the Departmental Office SS6018 (Print on it your NAME, STUDENT NUMBER, course number, and date.). And you must contact me to arrange a time within one week for an individual oral makeup test. A written-answer final exam (2 hours) will be given during the faculty exam period. The final exam accounts for 51% of the final mark.

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