Who is the fastest man in the world?

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Abstract

I compare the world record sprint races of Donovan Bailey and Michael Johnson in the 1996 Olympic games and try to answer the questions a) who is faster?, b) which performance was more remarkable?

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1 Introduction

At the 1996 Olympic summer games in Atlanta, both Donovan Bailey (Canada) and Michael Johnson (U.S) won gold medals in track and field. Bailey won the 100 metre race in 9.84 sec., while Johnson won the 200 metre race in 19.32 sec. Both marks were world records. After Johnson's race, an excited U.S television commentator "put Johnson's accomplishment into perspective" by pointing that his record time was less than twice that of Bailey's. Of course this is not a fair comparison, since the start is the slowest part of a sprint, and Johnson only had to start once, not twice.

Ato Bolton, the sprinter who finished third in both races, was also overwhelmed by Johnson's performance. He said that while normally the winner of the 100 metre race is considered the fastest man in the world, he thought that Johnson was the now fastest.

In this article I carry out some analyses of these two world record performances. I do not produce a definitive answer to the provocative question in the title, as that depends greatly on what one means by "fastest". Hopefully some light is shed on this interesting and fun debate. The results of the races are shown in Tables 1 and 2.

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2 Speed curves

A straightforward measure of a running performance is the speed achieved by the runner as a function of time. Table 3 gives the interval times for Bailey, obtained from Swiss Timing and reported in the Toronto Sun newspaper. These times were not recorded for Johnson. The value 7.7 is almost surely wrong, as it would imply an interval time of only 0.5 second for 10m.

The estimated times at each distance shown in Table 4 were obtained manually from a videotape of the races.

I contacted Swiss Timing about their possible error, and they rechecked their calculations. As it turned out, the split times were computed using a laser light placed 20m behind the starting blocks, and they had neglected to correct for this 20m gap in both the 70m and 80m split times.

Table 5 compares the estimated and official split times. After the 40m mark, the agreement is fairly good. The disagreement at 10, 20 and 30m is due to the paucity of data and the severe camera angle for that part of the race. Fortunately these points do not have a large influence on the results, as our error analysis later shows. Overall, this agreement gives us some confidence about the estimated times for Johnson, and some idea of the magnitude of their error. The speed curves were estimated by fitting a cubic smoothing spline to the first differences of the times, constraining the curves to be zero at the start of the race. The curves for each runner are shown in the top panel of Figure 1.

If Johnson's speed curve always lay above Bailey's, then this analysis would have provided convincing evidence in favour of Johnson since he achieved his speed despite having already run 100 metres. However Bailey's curve does rise above Johnson's, and achieves a higher maximum (13.2m/sec for Bailey, 11.8m/sec for Johnson). The 95% confidence interval for the difference between the maxima is (-.033, 2.81). Hence there is no definitive conclusion from this comparison.

We note that the 13.2m/sec differs from the figure of 12.1m/sec reported by Swiss timing. Bailey's estimated final speed is 12.4m/sec, versus 11.5m/sec reported by Swiss timing. This size of discrepancy is not unexpected since the interval times are only given to within 0.1 of a second. When a sprinter is running at top speed, he covers 10m in approximately 0.8 seconds, giving a speed of 10/.8=12.5 m/sec. Now if each of the interval times are off by 0.05s, then the estimated speed ranges from 10/.9=11.1 m/sec to 10/.7=14.3m/sec.

Who would win a race of say 150 metres? In essence this cannot be predicted because each runner would modify his strategy in an unknown way. Here is a simple-minded approach to the question. Bailey's speed at the 100m mark was 12.4 m/sec, and he was decelerating by an amount of only .036m/sec every 10m. Johnson's estimated time at 150m was 14.75 seconds. In order to beat that time, Bailey would need "only " to maintain an average speed of more than 10.2m/sec. Of course it's not clear whether he could do this.

For interest, in the bottom panel of Figure 1 Bailey's curve is compared to

Table 1: Results for 1996 Olympic 100m final

Name	Time	Reaction time
1. Bailey, Donovan (Canada)	9.84	+.174
2. Fredericks, Frank (Namibia)	9.89	+.143
3. Boldon, Ato (Tobago)	9.90	+.164
4. Mitchell, Dennis (USA)	9.99	+.145
5. Marsh, Michael (USA)	10.00	+.147
6. Ezinwa, Davidson (Nigeria)	10.14	+.157
7. Green, Michael (Jamaica)	10.16	+.169
8. Cristie, Linford (Great Britain)	DQ	

Wind Speed: +0.7 m/sec

Table 2: Results for 1996 Olympic 200m final

Name	Time	Time at 100m	Reaction time
1. Johnson, Michael (USA)	19.32	10.12	+.161
2. Fredericks, Frank (Namibia)	19.68	10.14	+.200
3. Boldon, Ato (Trinidad and Tobago)	19.80	10.18	+.208
4. Thompson, Obadele (Barbados)	20.14	?	+.202
5. Williams, Jeff (USA)	20.17	?	+.182
6. Gracia, Ivan (Cuba)	20.21	?	+.229
7. Stevens, Patrick (Belgium)	20.27	?	+.151
8. Marsh, Michael (USA)	20.48	?	+.167

Wind Speed: +0.4 m/sec

Table 3: Official times at given distances for Bailey

Distance(m)	0	10	20	30	40	50	60	70	80	90	100
Original time(s)	.174	1.9	3.1	4.1	4.9	5.6	6.5	7.7?	8.2	9.0	9.84
Corrected time(s)	.174	1.9	3.1	4.1	4.9	5.6	6.5	7.2	8.1	9.0	9.84

Table 4: Estimated times at given distances. Bailey starts at the 100m mark

Distance(m)	0	50	100	+12.9	+40.3	+49.4	+67.7	+76.9	+86.0	+100
Bailey:	.174			2.8	5.0	5.7	7.0	7.8	8.5	9.84
Johnson:	.161	6.3	10.12	11.4	14.0	14.8	16.2	17.0	17.8	19.32

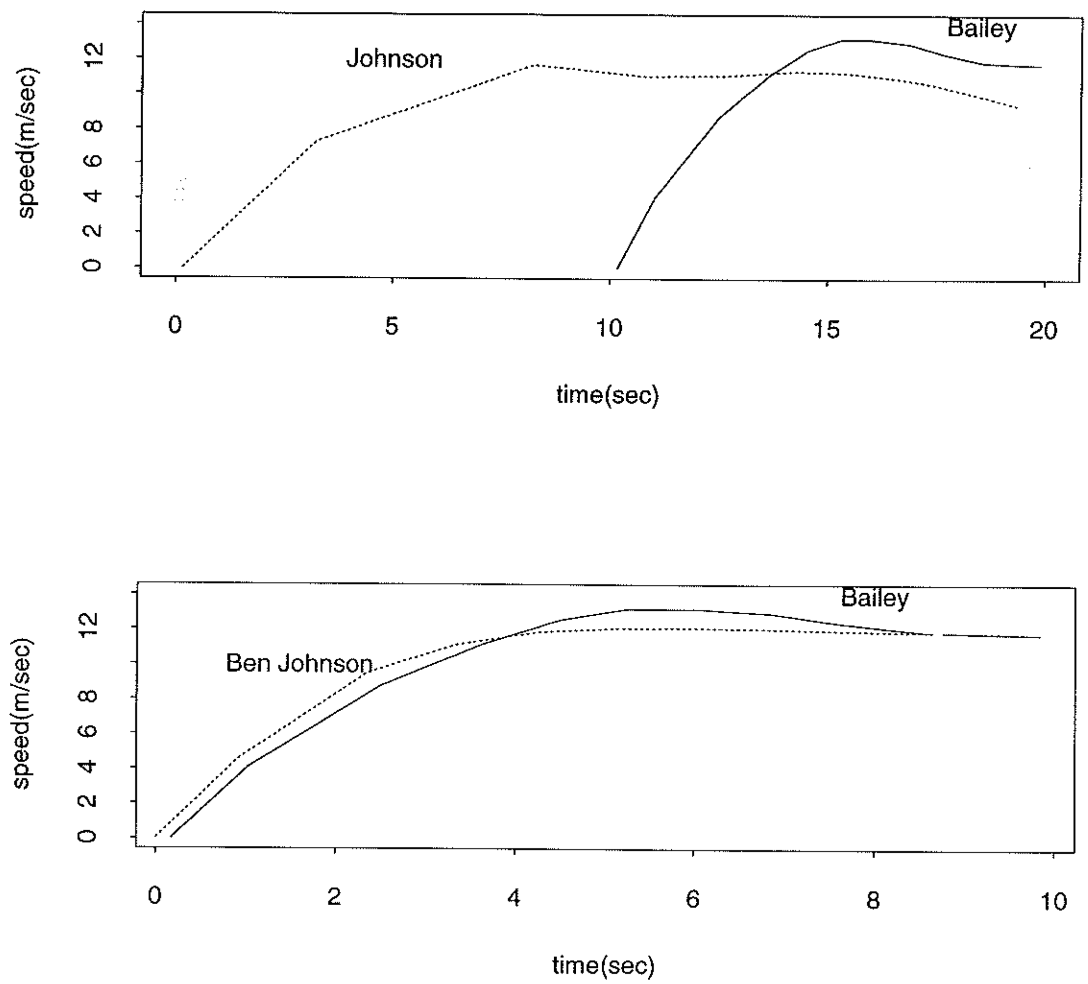

Figure 1: Top panel: estimated speed curves for Bailey and Johnson. Bailey's curves has been shifted to start at time 10.12 sec, Johnson's time at 100m. Bottom panel: estimated speed curves for Bailey and Ben Johnson, from the latter's 1987 world record race.

Table 5: Comparison of official and estimated interval times for Bailey

Distance(m)	0	10	20	30	40	50	60	70	80	90	100
Official	.174	1.9	3.1	4.1	4.9	5.6	6.5	7.2	8.1	9.0	9.84
Estimated	.174	2.1	3.4	4.3	5.1	5.7	6.4	7.2	8.0	8.9	9.84

Table 6: Estimated times (sec) for Johnson, for distances over 100 metres.

	100	110	120	130	140	150	160	170	180	190	200
Johnson	10.12	11.17	12.06	12.97	13.87	14.75	15.60	16.45	17.34	18.26	19.32

that from Ben Johnson's 1987 9.83s world record race (he was later disqualified for drug usage). They achieved roughly the same time in quite different ways: Ben Johnson got a fast start and then maintained his velocity; Bailey accelerated much more slowly but achieved a higher maximum speed.

3 Adjustment for the curve

Johnson's first 100m (10.12s) was run on a curve. In principal one should be able to adjust for the effect and predict what his time would have been on a straight track.

The centrepital acceleration running of an object moving at a velocity v around a circle of radius r is $a = v^2/r$. The radius of the circular part of the track is $100/\pi = 31.83m$. With Johnson's velocity ranging from 0 to 11.8/m.sec, his centrepital acceleration ranges from 0 to $4.37 m/sec^2$. Although that acceleration is at a right angle to the direction he is moving (and hence it doesn't do any work) he expends energy counteracting it. Unfortunately, just how much energy is expended seems difficult to measure.

4 Predictions from Keller's model

According to the theory of Keller (1973), the force $f(t)$ applied by a sprinter at time t may be written as

$$f(t) = \frac{dv(t)}{dt} + \frac{v(t)}{\tau} \tag{1}$$

This is just Newton's second law, where it is assumed that the resistance force per unit mass is $v(t)/\tau$. Keller estimated τ to be .892m/sec, from various races. Excellent overviews of Keller's work are given by Pritchard (1993) and Pritchard & Pritchard (1994).

Starting with assumption (1) and a model for energy storage and usage, Keller shows that the optimal strategy for a runner is to apply his maximum force F during the entire race, leading to a velocity curve

$$v(t) = F\tau(1 - e^{-t/\tau}) \tag{2}$$

Here τ is a damping coefficient, so that $v(t)/\tau$ is the resistance force per unit mass. This applies to races of less than 291 metres. For greater distances, there is a different optimal strategy.

Figure 2 shows the optimal speed curves for the 100m and 200m races, with Bailey's and Johnson's superimposed. We used least squares on the (time, distance) measurements to find the best values of τ and F for each runner: these were (1.74, 7.16) for Bailey and (1.69, 6.80) for Johnson.

The relationship between the time T and race distance D is given by

$$D = F\tau^2(T/\tau + e^{-T/\tau} - 1) \tag{3}$$

We can use this to predict the times for a 150m race; note that the reaction times must be included as well. The predictions are 13.97s (Bailey) and 15.00s (Johnson). The latter is close to the estimated time of 14.75s for Johnson at 150m, in table 6.

The same model also predicts a completely implausible 200m time of 17.99s for Bailey. One shortcoming of the mode is the fact that the velocity curve (2) never decreases, while observed velocity curves usually do. To rectify this, it seems reasonable to assume that a sprinter is unable to maintain his maximum force F over the entire race, but instead applies a force $F - c \cdot t$ for some $c \geq 0$. Using this in (1) leads to velocity and distance curves

$$\begin{aligned} v(t) &= k - ct\tau - ke^{-t/\tau} \\ D &= kt - ct^2/2 + \tau ke^{-t/\tau - 1} \end{aligned} \tag{4}$$

where $k = F\tau + \tau^2 c$. We fit this model to the observed distances, giving parameter estimates for (τ, F, c) of (2.39, 6.41, .20) and (2.06, 6.10, .05) for Bailey and Johnson respectively. Note that the estimated maximum force is greater for Bailey than Johnson, but decreases more quickly. Bailey also has a higher estimated resistance.

The estimated curves are shown in the bottom panels of Figure 2. The estimated 150m times from this model are 14.73s for Bailey and 14.82s Johnson. The latter is close to the estimated time of 14.75s at 150m, from table 6.

This estimated winning margin for Bailey is .09s, with a 95% confidence interval of (.03s, .22s). Figure 3 shows boxplots of difference in the predicted 150m times from 200 bootstrap replications (see section 8 for details). The model also does not capture a possible change of strategy by either runner, in a 150m race. This might result in different values for c or F.

From Keller's theory one can also predict world record times at various distances, as a function of F and τ. Keller fit his predicted world record times

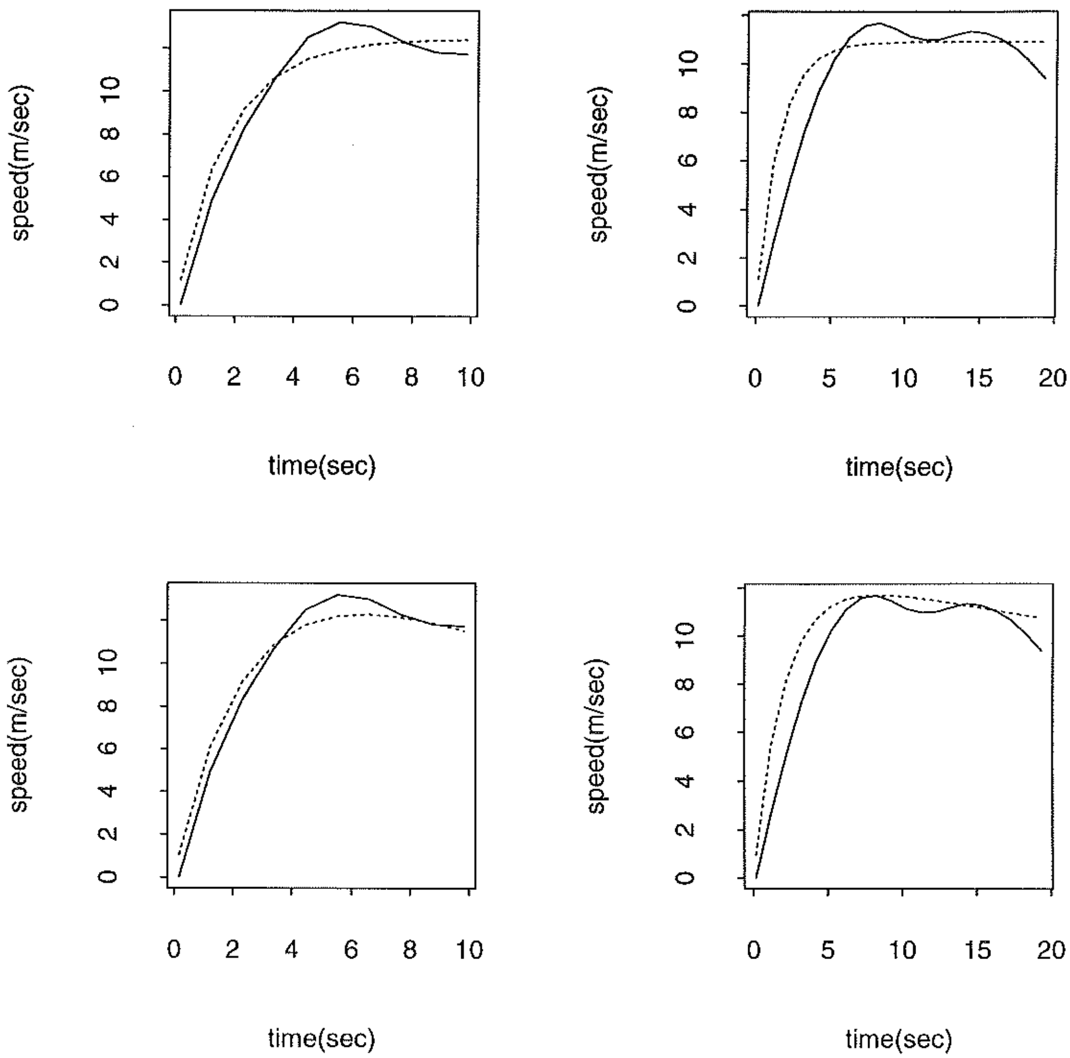

Figure 2: Optimal velocity curves (broken) for 100m (left panel) and 200m (right panel). The top row uses Keller's model (2); the bottom row uses the modified model (4). The solid curves are the estimated curves for Bailey and Johnson.

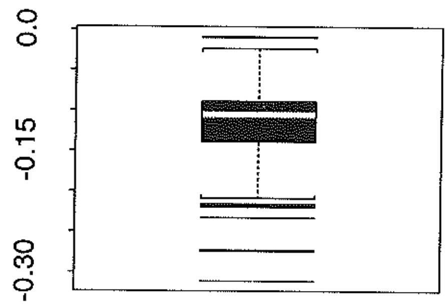

Figure 3: Boxplot of Bailey's minus Johnsons's predicted 150m times from 200 bootstrap replications

to the actual ones, for distance from 50 yards to 10000m, in 1973. From this he obtained the estimates $F = 12.2\text{m/sec}^2$, $\tau = 0.892\text{s}$. The fit was quite good[1] for 100m- 9.9s (actual), 10.07s (predicted); for 200m- 19.5s (actual), 19.25s (predicted). It is interesting that at the time, the 100m record was faster than expected while the 200m record was slower. Johnson's performance brings the 200m world record close to the predicted value. It may be that the 200m record has been a little "soft", with runners focussing on the more glamourous 100m race. Note that the predictions do not include a component for reaction time: with Johnson's reaction time of .161s, the predicted record would be 19.41s.

5 Comparison to other race competitors

In the rest of this article, I focus on the question of which of the two performances was more remarkable. These two races were particularly unique because the same two runners (Fredericks and Bolton) finished second and third in both. This suggests an interesting comparison. Figure 4 shows the percentage that each runner achieved as a function of the winning time in the race, for the

[1] The world records that Keller reports in 1973 of 9.9s and 19.5s are questionable. The 100m record was 9.95s, although 9.9s was the best hand-timed performance. The 200m record was 19.83s.

100 meters (solid curve) and 200 metre race (broken curve). Johnson's winning margin was particularly impressive. Also plotted on the figures are the corresponding percentages achieved in the previous 9 Olympic games, going back to 1952.[2] There has never been a winning margin as large as Johnson's in a 200 metre Olympic race, and only once before in a 100 metre race. This was Robert Hayes 10.05 sec. in 1964, versus 10 25 for the second place finisher. Johnson's margin over the second place Fredericks is also larger than the margin between the winner and *third place* in all but two of the races.

6 Evolution of the records

The top panels of Figure 5 show the evolution of the 100 and 200 metre records since 1950. The proportion improvements, relative to the existing record, are shown in the middle panels of Figure 5. Johnson's 19.32 performance represented the second largest percent improvement in the world record, the largest being Tommie Smith's lowering of the 20.32 world record to 20.00 in 1968. If we include Johnson's 19.66 world record in the 1996 U.S Olympic Trials, then overall he lowered Pietro Mennea 19.72 world record by 2.02% in 1966 (as of August), while Smith lowered the existing record 2.60%.

The bottom left panel of Figure 5 shows the ratio of the 200 meter world record versus the 100 meter world record, from 1950 onward. The ratio has hovered both above and below 2.0, with Johnson's world record moving it to an all time low of 1.963. The average speed in the 100m record race was 10.16 m/sec, while that for the 200m race is 10.35 m/sec, the fastest average speed of any of the sprint or distance races. This ratio of below 2.0 is predicted by the mathematical model of Keller (1973).

7 Conclusions

Who is faster, Bailey or Johnson? The answer depends on the definition of "faster", and there is no unique way of comparing two performances at different distances. Our results are inconclusive on this issue:

- it is not fair to compare the average speeds (higher for Johnson), since the start is the slowest part of the race and Johnson had to start only once.
- Bailey achieved a higher maximum speed; Johnson maintained a very high speed over a long time interval

[2]Throughout this analysis I restrict attention to post 1950 races, since before that time running in a straight or with one curve was not separated and it was not always known which way each race had been organized.

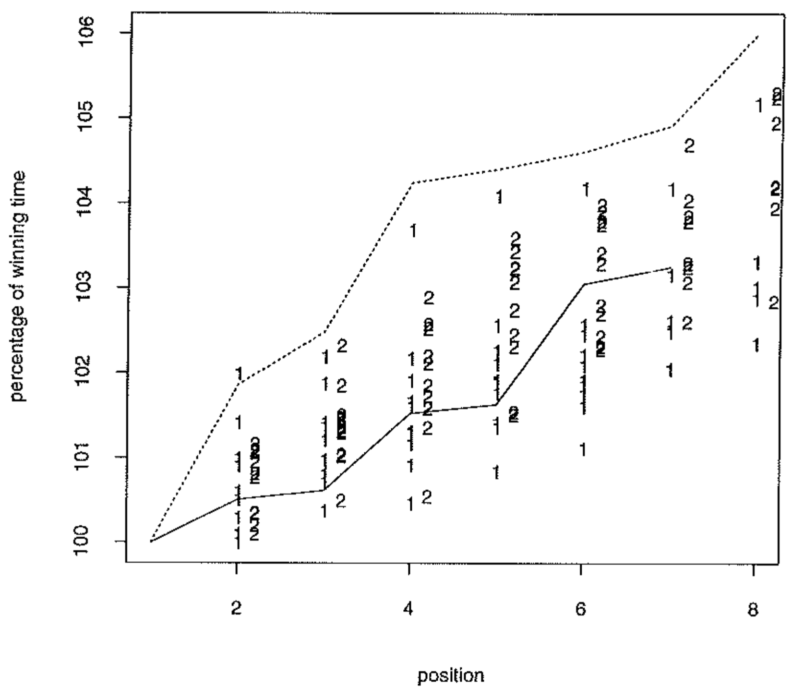

Figure 4: Percentage that each runner achieved as a function of the winning time in the race, for the 100 meters (solid curve) and 200 metre race (broken curve). The '1s' and '2s' correspond to the previous nine Olympic 100m and 200 m finals, respectively

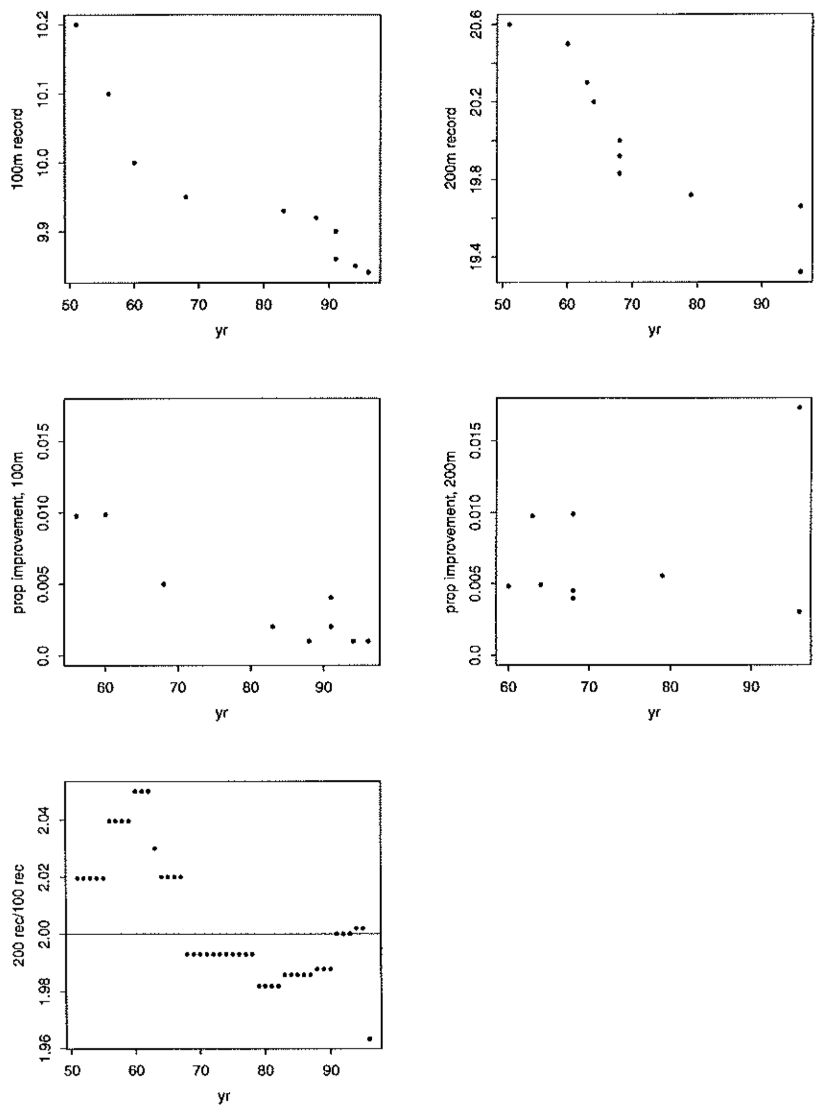

Figure 5: The top panels show the evolution of the 100m (left) and 200m (right) world records. The middle panels show the proportion improvement of the existing record that was achieved each time in the 100m (left) and 200m (right). The bottom left panel shows the evolution of the ratio of the 200m to 100m world record times.

- A simple analysis shows that if Bailey could have maintained an average speed over 10.2m/sec after 100m, he would have reached 150m before Johnson did. However it's not clear whether he could have done this.

- Predictions from an extended version of Keller's optimal running model suggest that Bailey would win a (straight) 150m race by .09s. However they do not account for the fact that Johnson's times are based on a curved initial 100m.

Whose performance was more remarkable? Here Johnson has the clear edge:

- Johnson's winning margin over the 2nd and 3rd place finishers (the same runners in both races!) was much larger than Bailey's, and was the second largest in any Olympic 100m or 200m final race in history.

- Johnson's improvement of the existing world record was the second largest in history for the 100m or 200m races.

- However the 200m record might have been a little "soft", since it was well above the record as predicted by Keller's theory.

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8 Appendix- error analysis

The data in table 4 were obtained manually from a videotape of the race, and hence are subject to measurement error. To assess the effect of this error, we applied the parametric bootstrap (see Efron & Tibshirani (1993)). The maximum amount of error in the times was thought to be around $\pm .05s$ for Bailey's times, and $\pm$.20s for Johnson's early times and $\pm$.15s for Johnson's last 100m times Therefore I added uniform noise on these ranges to each time measurement. For each resulting dataset I estimated the speed curves for Bailey and Johnson. This process was repeated 200 times.

The observed difference in the maximum speeds was 12.9-11.7=1.2m/sec. The upper and lower 2.5% points of the 200 observed differences was (-.44, .71). Bailey's speed estimates had an average standard deviation of .13, while Johnson's had and average standard deviation of .34.

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