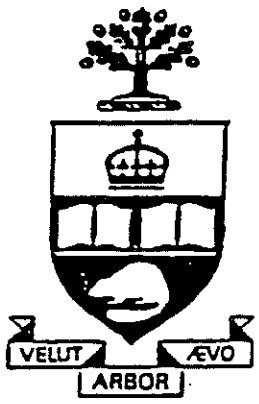

Measuring Warehouse Performance

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Measuring Warehouse Performance

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Abstract

This report presents a methodology for the measurement of warehouse performance.

1 Introduction.

A producer engages a warehouse to store and deliver a collection of products. The producer maintains records of the amount of each product assigned to the warehouse and the amount of each product to be delivered to customers.

Let $a_{i,t}$ denote the amount of product i assigned to the warehouse on date t. Let $d_{i,t}$ be the amount of product i to be delivered to customers on date t.

The cumulative total of all product i assigned to the warehouse up time t is denoted by $A_{i,t}$. The cumulative total of all product i to be delivered by the warehouse up time t is denoted by $D_{i,t}$.

As a first approximation one might expect the warehouse to hold the difference $W_{i,t} = A_{i,t} - D_{i,t}$.

A survey can determine the actual amount of product i in the warehouse at time t, $C_{i,t}$

The observed count C may differ from the the predicted count W for many reasons including:

- Over/Under stocking. More or less product arrived than was represented by the producer record, $a_{i,t}$.
- Over/Under shipping. More or less product was shipped than was represented by the producer record, $d_{i,t}$.
- Damage and loss. Product was lost in the path to the warehouse or during the period in the warehouse.

2 Purpose

We seek a measure of warehouse performance. The measure should have the following characteristics:

1. Validity. The measure should assess those activities which are the responsibility of the warehouse.
2. Reliability. Repeated measurement should yield essentially the same result.
3. Credibility. The measure should be accepted as valid by all users.
4. Efficiency. The measure should be as fast and inexpensive as possible given the above constraints.

3 Data currently used

Data is currently available from three sources:

- Records maintained by the producer of products assigned to and from the warehouse,
- Records maintained by the warehouse of products received and shipped from the warehouse, and
- Counts of inventory in the warehouse for a sample of products.

4 Proposed Data

The data currently used can only assess discrepancies between records maintained by the supplier and those associated with the warehouse. These discrepancies therefore reflect the performance of the producer's accounting system *and* operation of the warehouse. In order to measure the performance of the warehouse it is necessary that a system be developed to accurately reflect the flow of products to and from the warehouse.

It is not the purpose of this paper to make specific recommendations. We have in mind a mechanism by which both parties, at the point of transfer of product from the producer to the warehouse agree on the nature and amount of the transfer. (Signed receipts are a traditional method of recording such transactions. A modern version would perhaps involve electronic reading of bar codes.) The important feature of the method should be that both parties participate in the acquisition of the data.

The cumulative history of such transactions forms the basis of assessing the amount of product moving to the warehouse. Similarly the cumulative history of shipping directives forms the basis of assessing the amount of product shipped from the warehouse. It is essential that both parties trust the accuracy of these histories.

The assessment of product in the warehouse should be done by sampling products. The only way to draw a representative sample is to draw a random sample of products. The amount of each of the selected products would then be determined by counts made in the warehouse.

4.1 Sampling

4.1.1 Purpose

To estimate the total loss associated with the discrepancy between administrative records and warehouse contents.

4.1.2 Population

The population is the total amount of product in the warehouse according to the administrative records. The sample frame is the list of all products for which some amount should be in the warehouse.

4.1.3 Sample Unit

The warehouse contains a variety of products. The sampling will involve counting the amount of a selection of products. The sampling unit is the particular product.

4.1.4 Stratified sampling

The warehouse will have products of differing values and differing amounts. Since we are forming an aggregate measure, it will be efficient to sample more heavily products which are more costly and which are in greater abundance.

We propose to stratify the product lines according to the value of the product line, $W_i \times V_i$. Within each strata, we propose to sample according to simple random sampling. The strata consisting of more valuable product lines will be sampled more heavily, in order to increase their contribution to the estimate of warehouse loss. This design would reflect the producer's greater concern with high value product lines. Alternatively, the producer and the warehouse manager could agree to a variety of sampling schemes, so as to sample more heavily from product lines that were in higher demand from consumers.

4.1.5 Loss measure

The discrepancy $C_i - W_i$ may be associated with a loss. We believe that the loss should be related to the value V_i, the amount that should be present, W_i and the discrepancy $C_i - W_i$. The loss function currently used is a simple function only of the fraction of the discrepancy: $(C_i - W_i)/W_i$. No account is taken of the sign of the discrepancy (positive or negative) nor of the amount W_i nor of the value V_i.

We believe that it would be useful to develop one or more loss functions which reflect more usefully the loss associated with any discrepancy. We discuss some possible functions. These functions differ only in the form of the loss. Products may be grouped and a form of loss assigned to each group. The producer would be responsible for grouping the products and selecting the form of the loss function. Possible loss functions might include:

- Loss = the absolute value of the discrepancy.

- Loss = the absolute value of the discrepancy, if the discrepancy is positive, and half of the absolute value of the discrepancy, if the discrepancy is negative. This loss function attaches a greater penalty to having fewer items in stock than records indicate.

- Loss = the absolute value of the discrepancy plus some penalty function, if the discrepancy is positive, and half of the absolute value of the discrepancy, if the discrepancy is negative. The penalty function increases the loss function, by adding an additional penalty for annoyance to the producer. A suggested penalty function could be the minimum of ten units of the product, or of the value of the product line.

4.1.6 Analysis

Let the total loss associated with sampled product i be denoted by L_i. We assume that there were N product lines in the warehouse. Assume that a sample size of n was drawn, with n_i product lines sampled in strata i. Let $\overline{l_i}$ denote the sample average of the loss in the i^{th} strata. Then $\sum_{i=1}^{k} N_i \overline{l_i}/N$ estimates the average loss per product line in the warehouse, where N_i is the number of product lines in strata i. One can then estimate the percent loss as a proportion of warehouse value by $100 \times (\sum_{i=1}^{k} N_i \overline{l_i})/(\sum_{i=1}^{k} W_i V_i)$.

4.1.7 Sample Size

The size of the sample will affect the precision of the estimate. The variance of the estimate of average loss is given by $\frac{1}{N^2} \sum_{i=1}^{k} N_i^2 (\frac{N_i - n_i}{N_i}) \frac{\sigma_i^2}{n_i}$, where σ_i^2 is the variance of the loss in strata i. If these σ_i are approximated, then the strata sizes may be chosen to minimize this approximation.

5 Summary

We believe the measure and sampling is

- Valid. It measures performance in real terms.

- Reliable. The random sample produces estimates with known precision.

- Credible. It is based on agreed data.
- Efficient. The weighted sampling plan produces estimates with the greatest precision for the amount of effort.

6 Example

In the Appendix, data for seventy-one product lines was provided by Thomson Terminals Limited, for a demonstration of these principles.

A loss function was developed that was equal to the absolute value of the value of the discrepancy plus a penalty function, if the discrepancy is positive, and half of the absolute value of the discrepancy, if the discrepancy is negative, as shown in figure 1. The penalty function was the minimum of the value of ten units of the product, or of the value of the product line. We investigated two samples sizes 23 and 47. The product lines were divided into three strata according to value, as shown in figure 2. The first strata consisted of the 7 most valuable product lines, the second strata consisted of the next 20 most valuable lines, and the third strata consisted of the remaining product lines.

The sample sizes for each strata were roughly chosen to minimize the estimated variance $\frac{1}{N^2}\sum_{i=1}^{k} N_i^2(\frac{N_i-n_i}{N_i})\frac{\sigma_i^2}{n_i}$ where the σ_i^2 are approximate by the sample within strata variance. The sample sizes within each strata are described in the following table.

Sample Size	Strata 1	Strata 2	Strata 3
23	7	10	6
47	7	20	20

The sampling process was simulated 200 times for each sample size and histograms of the estimates prepared. These displays indicate the precision to be expected under the two sizes of sampling. When samples of size 23 were drawn two hundred times, (with a loss function as in Figure 1, with penalty equal to the value of ten units, or to the value of the product line, whichever was less), the mean loss per product line was $650.73. The mean percent loss as a proportion of warehouse value was 1.63%. When two hundred samples of size 47 were drawn (with loss function as above), the mean loss per product

line was $640.99. The mean percent loss as a proportion of warehouse value was 1.60%. The actual mean loss per product line was $645.31. The actual loss as a proportion of warehouse value was 1.61%.

7 Appendix

Product Number	Book Count	Actual Count	Unit Cost
1	3481	3471	12.263
2	14451	13609	14.509
3	5689	6547	14.449
4	4865	4862	16.273
5	2468	2482	16.266
6	13039	13083	13.945
7	4019	4096	13.939
8	148	5	9.649
9	8212	8380	10.415
10	6314	6297	10.392
11	9107	9147	13.74
12	3645	3520	13.733
13	23	23	628.265
14	2537	2531	14.857
15	2647	2573	14.852
16	542	546	10.477
17	139	137	11.508
18	3383	3392	11.499
19	379	379	10.914
20	24573	24424	10.905
21	17110	17019	8.397
22	2970	3046	16.595
23	1428	1425	13.26
24	13126	13090	11.427
25	16679	16582	10.888
26	11374	11336	8.404
27	6657	6716	10.703
28	3060	3075	10.669
29	2478	2435	9.093
30	10	0	14.755
31	2002	1962	14.142
32	1897	1741	14.149
33	1317	1368	12.552
34	18	0	15.875
35	1265	1193	8.189

Product Number	Book Count	Actual Count	Unit Cost
36	427	424	8.712
37	7045	7040	10.753
38	1339	1336	11.3
39	21	0	12.043
40	1321	1322	12.281
41	2063	2095	14.228
42	2639	2646	12.034
43	1413	1411	13.197
44	2450	2394	14.474
45	6	0	16.835
46	3	3	14.219
47	3580	3573	12.035
48	1512	1494	14.221
49	4624	4587	12.027
50	2286	2268	13.19
51	880	856	14.468
52	1430	1391	18.196
53	866	865	18.189
54	698	696	17.551
55	370	367	14.292
56	648	644	11.951
57	67	11	16.593
58	111	45	11.064
59	27	30	11.833
60	1195	1195	8.825
61	982	977	14.509
62	432	432	14.449
63	2938	2938	13.733
64	80	80	14.852
65	70	70	8.397
66	144	144	16.595
67	157	157	11.427
68	694	694	10.888
69	377	377	12.281
70	400	400	18.189
71	453	453	14.292

Loss Function X where X=discrepancy + p, if discrepancy is positive, and X=discrepancy/2 if discrepancy is negative

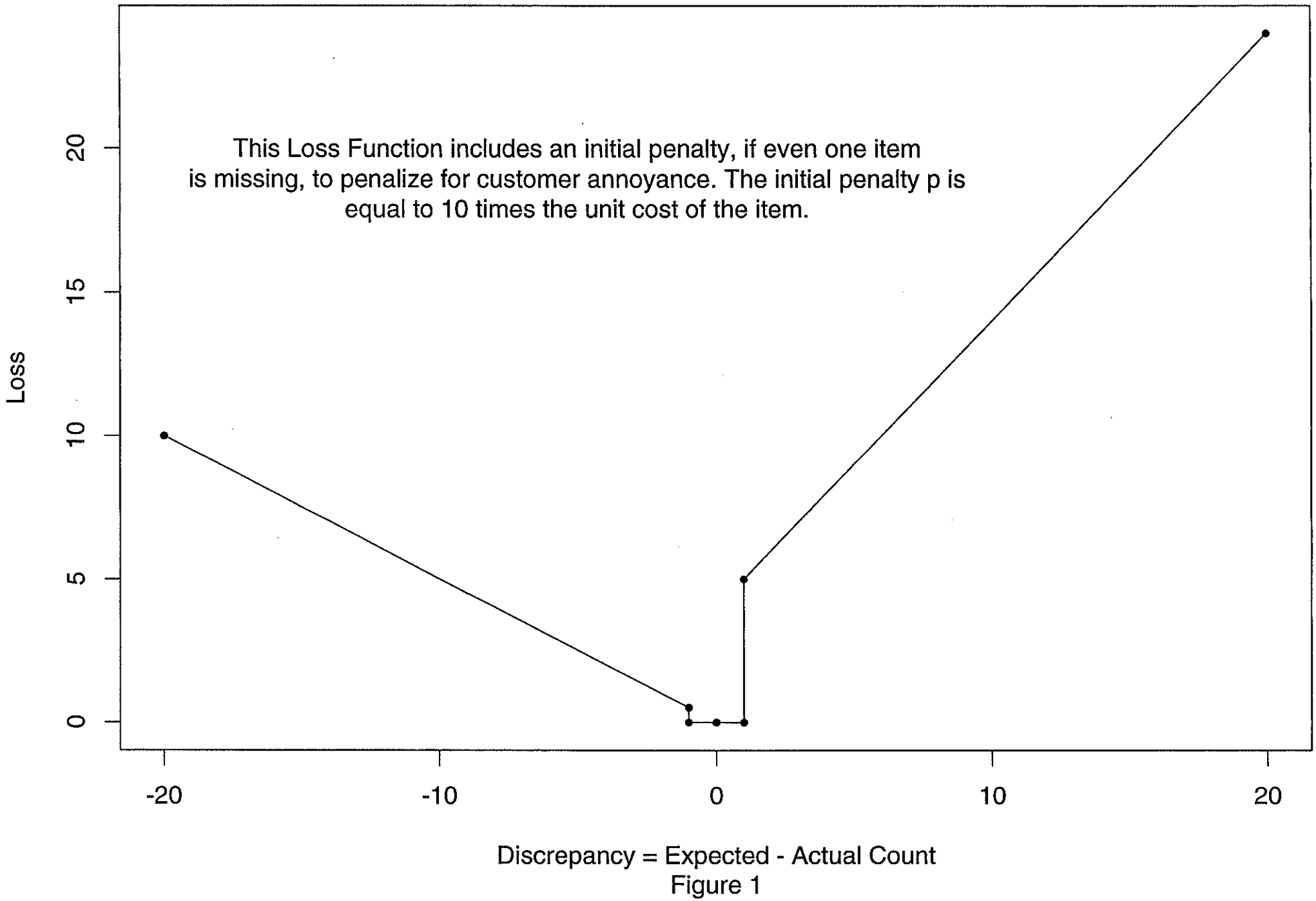
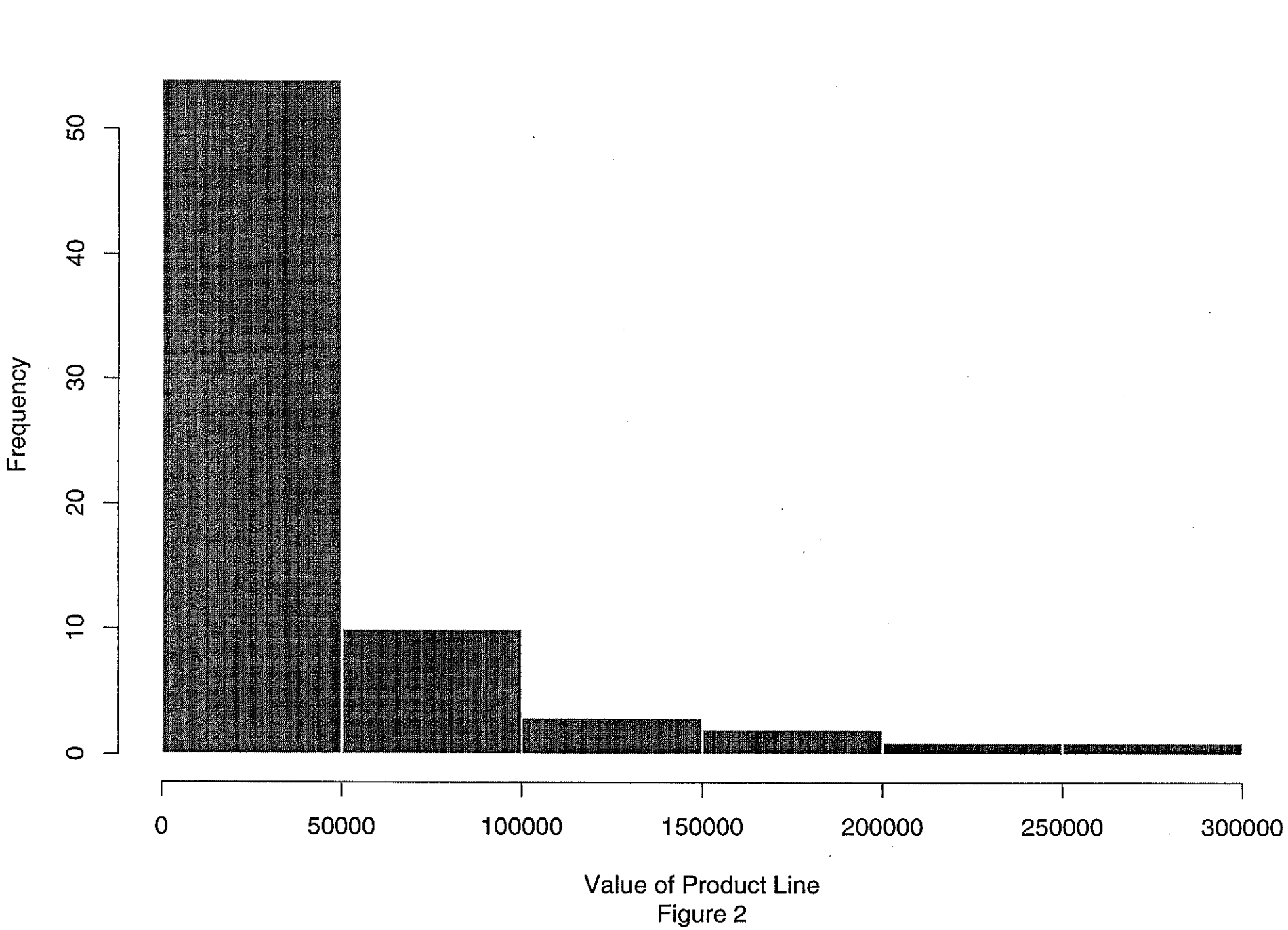
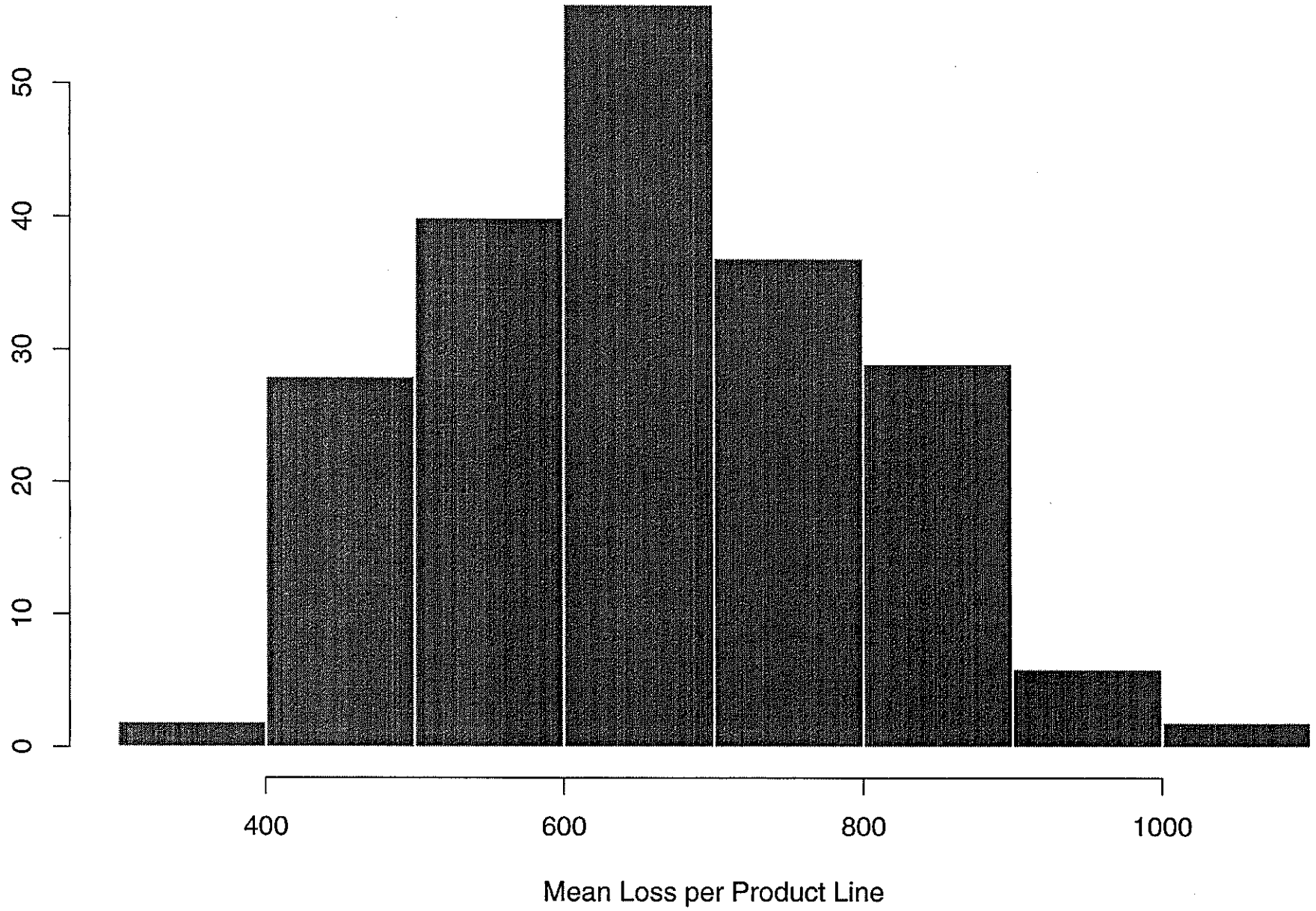
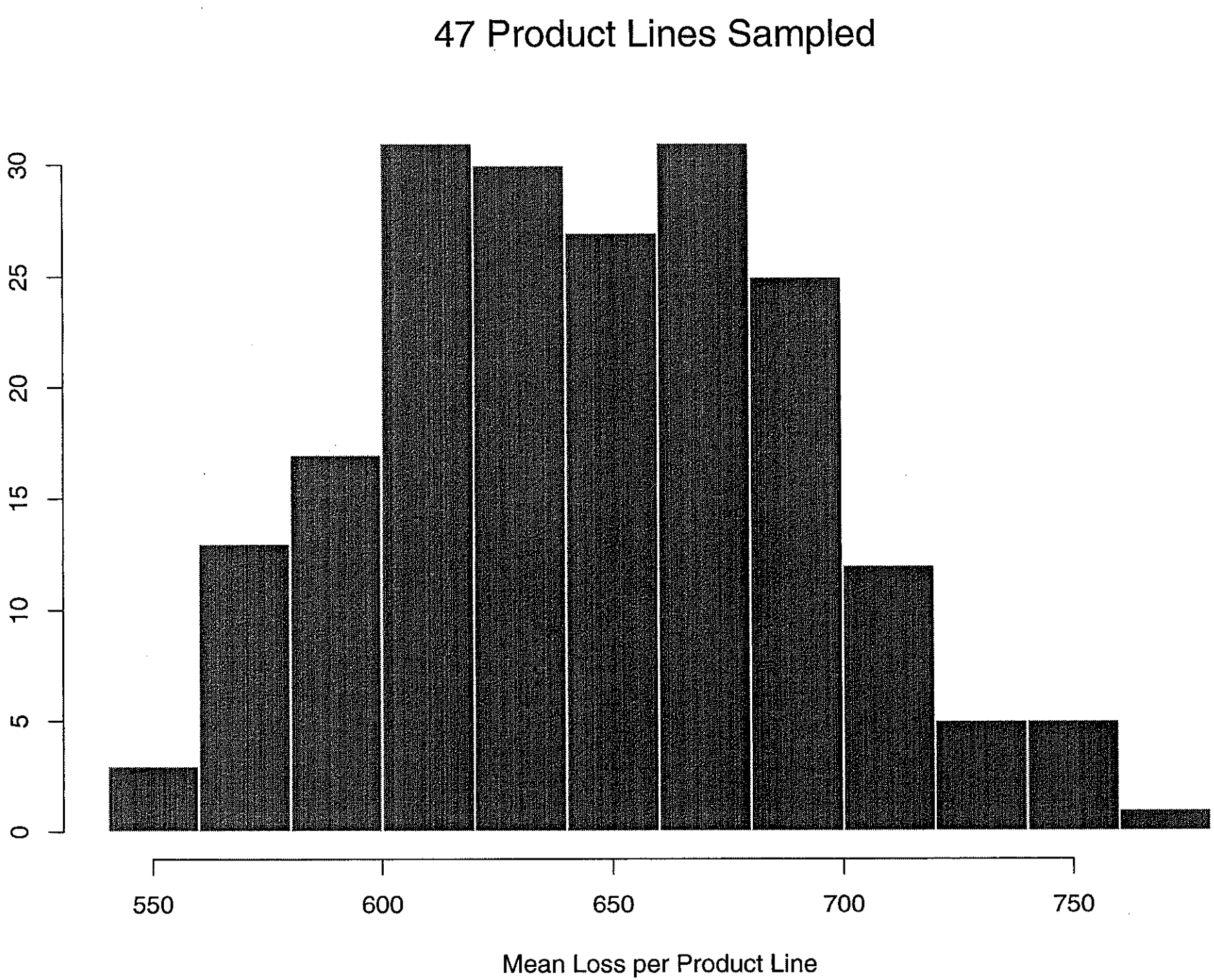


Figure 1


Histogram of Values of the Product Lines
Frequency
0
10
20
30
40
50
0
50000
100000
150000
200000
250000
300000
Value of Product Line
Figure 2


23 Product Lines Sampled
0
10
20
30
40
50
400
600
800
1000
Mean Loss per Product Line


47 Product Lines Sampled
0
5
10
15
20
25
30
550
600
650
700
750
Mean Loss per Product Line