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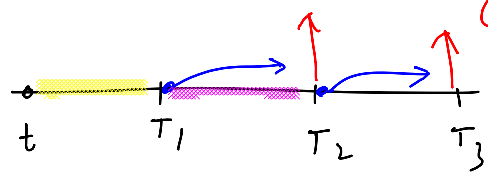
A Cap is a collection of caplets:

$$Cp_t^{(N)} = \sum_{k=1}^{N} Cpl_t^{(k)} \qquad \text{cap of mat } T_N$$



$(l_{T_2}^{(2)} - K)_+ \; N \Delta T_2$

$\shortparallel$

$T_3 - T_2$

$$Cpl_t^{(k)} = P_t^{(k+1)} \left(l_t^{(k)} \, \Phi(d_+) - K \, \Phi(d_-) \right)$$

$$d_\pm = \frac{\ln(l_t^{(k)}/K) \pm \frac{1}{2}\sigma_{eff}^2}{\sigma_{eff}}$$

$$\sigma_{eff}^2 = \int_t^{T_k} (\sigma_s^{(k)})^2 \, ds$$

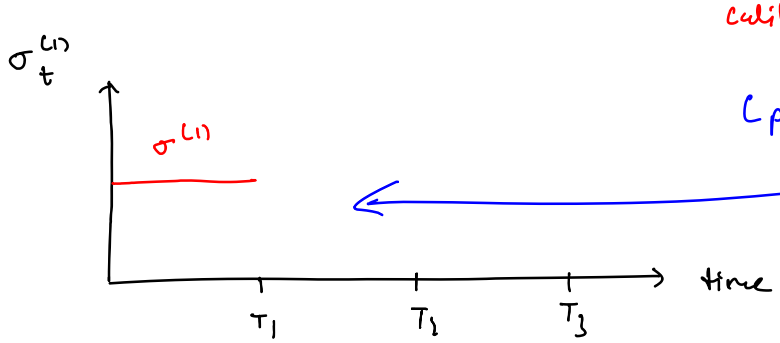
need

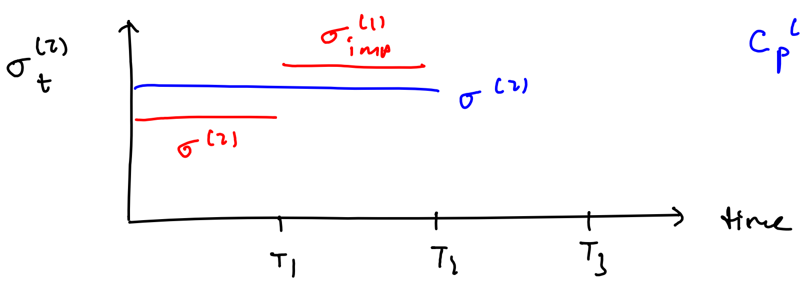
$\int_0^{T_1} (\sigma_s^{(1)})^2 \, ds$ For $Cpl^{(1)}$

$\int_0^{T_2} (\sigma_s^{(2)})^2 \, ds$ For $Cpl^{(2)}$

calibrate to $Cp^{(1)}$

$Cp^{(1)} = Cpl^{(1)}(\sigma_{imp}^{(1)})$





$Cp^{(2)} = Cpl^{(1)}(\sigma_{imp}^{(1)})$

$+ \; Cpl^{(2)}(\underbrace{\sigma', \sigma''}_{\to\, \sigma^{(2)}})$

	$[0, T_1]$	$(T_1, T_2]$	$(T_2, T_3]$	$\cdots$	
$\ell^{(1)}$	$\sigma^{(1)}$	–	–	$\rightarrow$	$\sigma^{(1)}_{imp}$
$\ell^{(2)}$	$\sigma^{(2)}$	$\sigma^{(1)}$	–		$\sigma^{(2)}_{imp}$
$\ell^{(3)}$	$\sigma^{(3)}$	$\sigma^{(2)}$	$\sigma^{(1)}$		$\sigma^{(3)}_{imp}$
$\ell^{(4)}$	$\sigma^{(4)}$	$\sigma^{(3)}$	$\sigma^{(2)}$		

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1. Calib LFM

2. sim LFM $\longrightarrow$ swaption price = LSM price (σ^F)

$\downarrow$

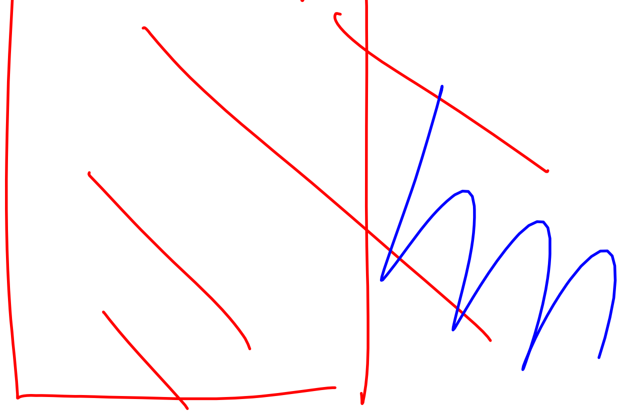
σ^F_{imp}

$\uparrow$

" true "

$\sigma^{F,1}_{imp} \neq \sigma^{F,2}_{imp}$

$(0, T_1)$	(T_1, T_2)
$\sigma^{F,1}_{imp}$	
$\sigma^{F,2}_{imp}$	$\sigma^{F,1}_{imp}$



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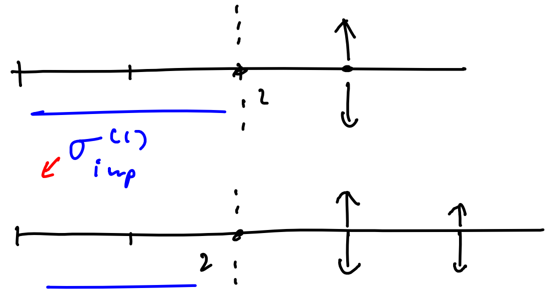
$\mathrm{Swp}^{(N)}$: $\mathrm{Swp}^{(1)}$



$\eta \swarrow \sigma^{(1)}_{imp}$

$\mathrm{Swp}^{(2)}$

$\eta \swarrow \sigma^{(2)}_{imp}$

$$\frac{dF_t^{(N)}}{F_t^{(N)}} = \overset{\eta \uparrow}{\sigma_t^{(N)}} \, dW_t^A$$

want these to be consistent with prices from LFM sim.

$$\mathrm{SwpLFM}^{(k)} = \mathrm{SwpLSM}^{(k)}\left(\underset{\downarrow \, \eta}{\sigma^{(k)}_{imp}} \right)$$

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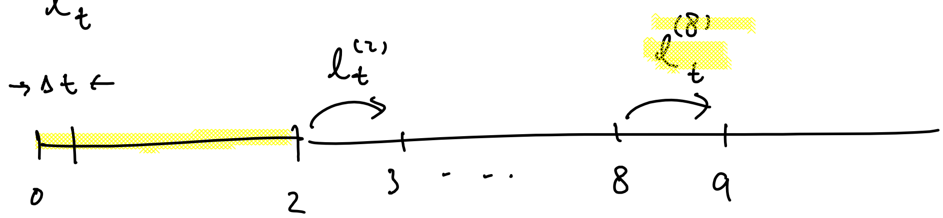
$$\frac{d\,l_t^{(k)}}{l_t^{(k)}} = \sum_{n=1}^{k} \frac{(\cdot)(l_t^{(n)})}{(\cdot)(l_t^{(n)})}\,dt + \sigma_t^{(k)}\,dW_t^{(2)}$$



$$\frac{d\,l_t^{(2)}}{l_t^{(2)}} = \underbrace{\frac{(\cdot)(l_t^{(2)})}{(\cdot)(l_t^{(2)})}}_{\mu_t^{(2)}}\,dt + \sigma_t^{(2)}\,dW_t^{(2)}$$

assume constant on $(t,\ t+\Delta t]$

$$l_{t+\Delta t}^{(2)} = l_t^{(2)} + \mu_t^{(2)}\,\Delta t\, l_t^{(2)} + \sigma_t^{(2)}\, l_t^{(2)} \sqrt{\Delta t}\, Z$$

$$Z \underset{Q^{(2)}}{\sim} N(0,1)$$

$$= l_t^{(2)} \exp\left\{ \left(\mu_t^{(2)} - \tfrac{1}{2}(\sigma_t^{(2)})^2\right)\Delta t + \sqrt{\Delta t}\,\sigma_t^{(2)}\, Z \right\}$$

$\longrightarrow$

$$l_2^{(2)},\ l_2^{(3)},\ \ldots,\ l_2^{(9)} \quad @\ T=2$$

$$\left.\begin{aligned} P_2^{(3)} &= (1 + \Delta T\, l_2^{(2)})^{-1} \\ P_2^{(4)} &= P_2^{(3)}\,(1 + \Delta T\, l_2^{(3)})^{-1} \\ &\vdots \\ P_2^{(10)} & \end{aligned}\right\} \quad @\ T=2$$

$$F_2 = \frac{P_2^{(2)} - P_2^{(10)}}{\Delta T \sum_{n=3}^{10} P_2^{(n)}}$$

$$Q = (F_2 - K)_+ \cdot N \cdot \Delta T \cdot \sum_{n=3}^{10} P_2^{(n)}$$

$$\frac{V_t}{P_t^{(2)}} = \mathbb{E}^{Q^{(2)}}\left[\frac{Q}{P_2^{(2)}}\right]$$

$$\Rightarrow \quad V_0 = P_0^{(2)} \, \mathbb{E}^{Q^{(2)}}[\,Q\,]$$

$$\approx P_0^{(2)} \frac{1}{M} \sum_{N=1}^{M} Q_N$$

$$\frac{d\,l_t^{(k)}}{l_t^{(k)}} = \sigma_t^{(k)}\, dW_t^{(k+1)}$$

Poisson $\quad \lambda^{(k+1)}$ - hazard rate

$$\longrightarrow \sigma_t^{(k)}\, dW_t^{(k+1)} + a_t\left(dN_t - \lambda\, dt\right)$$

$$\longrightarrow \sigma_t^{(k)}\, dW_t^{(k+1)} + a_t\left(dL_t - \lambda\, \mathbb{E}[l_1]\, dt\right)$$

compound Poisson