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$\leftarrow X^1_t$

$$\lambda^1_t = a_1 X^1_t + b_1$$

$$\lambda^2_t = a_2 X^1_t + b_2$$

$$\vdots$$

$$\lambda^N_t = a_N X^1_t + b_N$$

$$dX^1_t = \kappa(\theta - X^1_t)\,dt + \sigma\sqrt{X^1_t}\,dW_t$$

e.g.

$$\lambda^1_t = X_t + b$$

$$\lambda^2_t = a X_t + b$$

$$dX_t = -\kappa X_t\,dt + \sigma\,dW_t$$

$$q_1(T) = \mathbb{Q}(\tau_1 > T) = \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \lambda^1_s\,ds}\right]$$

$$= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T X_s\,ds}\right] e^{-bT}$$

$$= e^{A(T) - B(T)X_0}\, e^{-bT}$$

$$q_2(T) = \mathbb{Q}(\tau_2 > T) = \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \lambda^2_s\,ds}\right] \qquad Y_t = aX_t$$

$$= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T Y_s\,ds}\right] e^{-bT}$$

$$d(aX_t) = -\kappa(aX_t)\,dt + a\sigma\,dW_t$$

$$dY_t = -\kappa Y_t\, dt + (a\sigma)\, dW_t$$

$$q_2(T) = e^{A'(T) - B'(T)X_0 - \delta T}$$

$$q_{12}(T) = \mathbb{Q}(\tau_1 > T,\ \tau_2 > T)$$

$$= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T (\lambda_s^1 + \lambda_s^2)\, ds} \right]$$

$$= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T ((a+1)X_s + 2\delta)\, ds} \right]$$

$$= e^{-2\delta T}\ \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T Z_s\, ds} \right]$$

$$dZ_t = -\kappa Z_t\, dt + ((a+1)\sigma)\, dW_t$$

$$q_{12}(T) = e^{-2\delta T}\ e^{\underline{A''(T)} - \underline{B''(T)} X_0}$$

$$q_1(T)\, q_2(T) = e^{-2\delta T}\ e^{\underline{A'(T) + A(T)} - (\underline{B'(T) + B(T)}) X_0}$$

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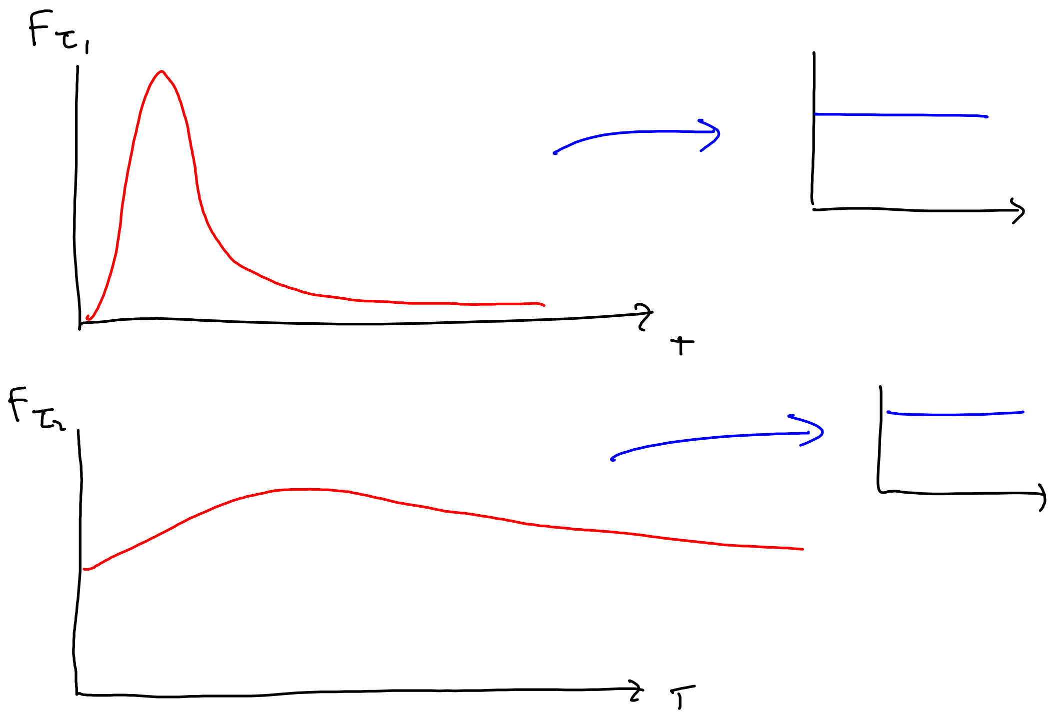
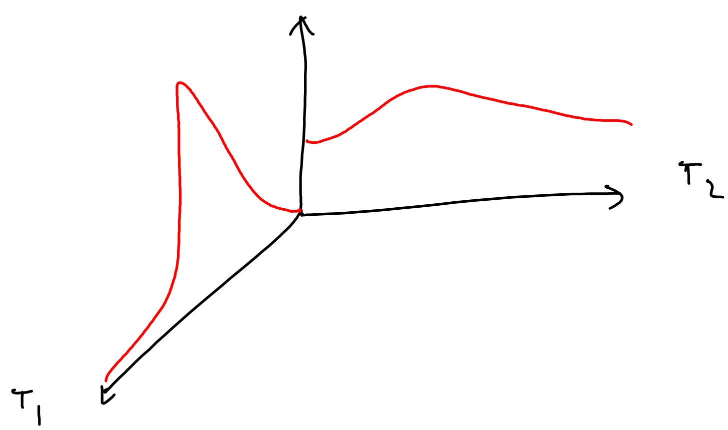




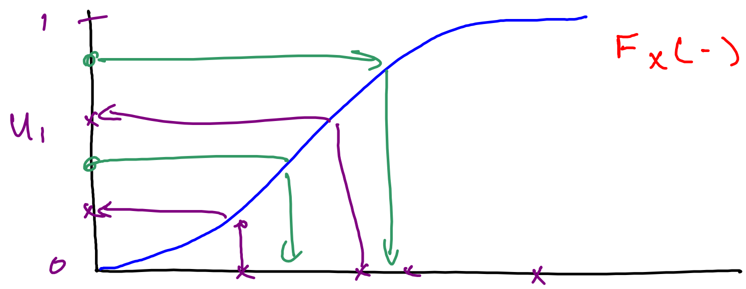
$$U_1 = F_X(X)$$

$$\mathbb{P}(U_1 < u) = \mathbb{P}(F_X(X) < u)$$

$$= \mathbb{P}(X < F_X^{-1}(u))$$

$$= F_X (F_X^{-1}(u))$$

$$= u. \qquad \Rightarrow U_1 \text{ is uniform } [0,1].$$



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$$C(u_1, u_2) \equiv \mathbb{P}(U_1 < u_1, U_2 < u_2)$$

$$C(F_X(x), F_Y(y)) = \mathbb{P}(U_1 < F_X(x), U_2 < F_Y(y))$$

$$= \mathbb{P}(F_X^{-1}(U_1) < x, F_Y^{-1}(U_2) < y)$$

$$= \mathbb{P}(X < x, Y < y)$$

$y \to +\infty$, $\quad \mathbb{P}(X < x, Y < y) \to \mathbb{P}(X < x)$

$$F_Y(y) \to 1$$

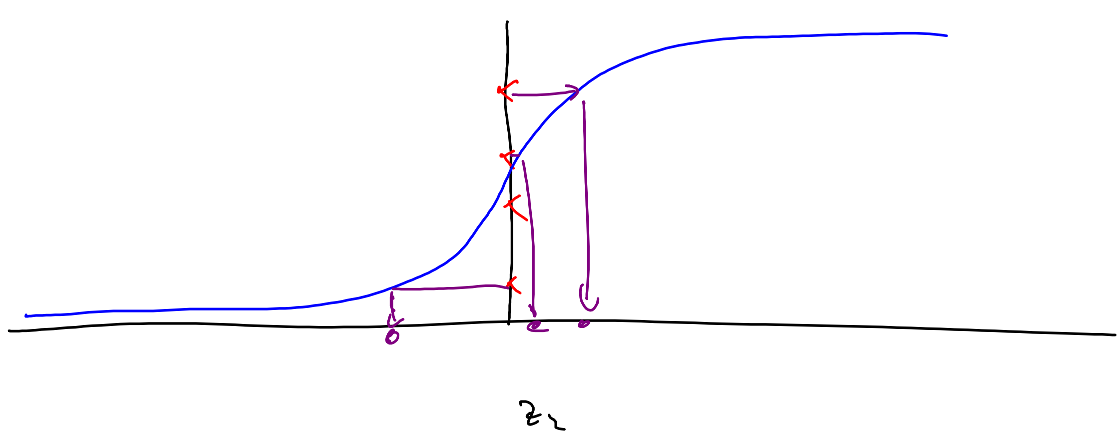
& $\quad C(u_1, 1) = \mathbb{P}(U_1 < u_1, U_2 < 1)$

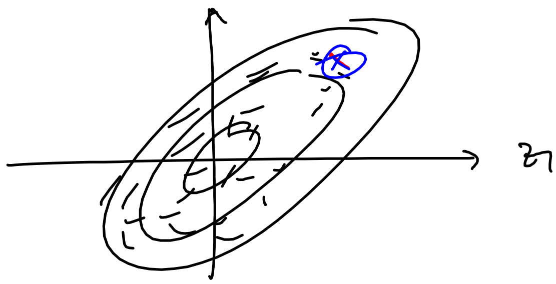
$$= \mathbb{P}(U_1 < u_1) = u_1$$

$\Rightarrow \quad \mathbb{P}(X < x, Y < y) \to C(F_X(x), 1)$

$$= F_X(x).$$

$C(u_1, u_2) = u_1 u_2 \quad$ independence copula.





$$(z_1, z_2) \sim \Phi_{\Sigma}(\cdot, \cdot)$$

$$z_1 \longrightarrow u_1 = \Phi(z_1) \longrightarrow \tau_1 = F_{\tau_1}^{-1}(u_1)$$

$$z_2 \longrightarrow u_2 = \Phi(z_2) \longrightarrow \tau_2 = F_{\tau_2}^{-1}(u_2)$$

$$C(U_1 < u_1, U_2 < u_2, \ldots, U_n < u_n)$$

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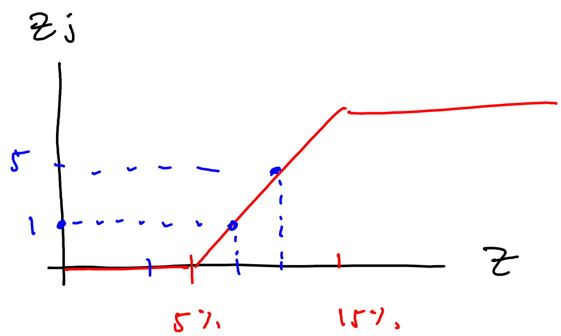
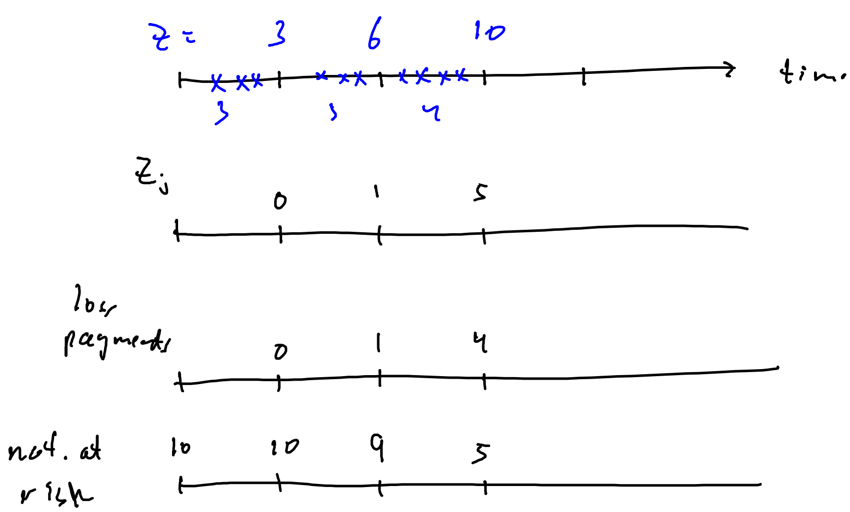
$$\frac{100}{1.05}\left[0.10 \times 0.20 + 0.9 \times 1\right] = 87.62$$

$$= \frac{100}{1+y}$$



N=100



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market factor ↓ ; idiosyncratic factors ↙

$$\tau_1 \to X_1 = a\,M + Z_1\,(1-a^2)^{1/2}$$

$$\tau_2 \to X_2 = a\,M + Z_2\,(1-a^2)^{1/2}$$

$$\vdots$$

$$\tau_n \to X_n = a\,M + Z_n\,(1-a^2)^{1/2}$$

$M \sim N(0,1)$
$Z_i \sim N(0,1)$
$a = \sqrt{\rho}$ to have correlation of ρ.

find x s.t.

$$\Phi(x) = \mathbb{P}(X_1 < x) = \mathbb{P}(\tau_1 < t) = F_1(t)$$

$$\Rightarrow x = \Phi^{-1}(F_1(t))$$

$$\mathbb{P}(\tau_1 < t, \tau_2 < t, \ldots, \tau_n < t \mid M)$$

$$= \mathbb{P}(X_1 < \Phi^{-1}(F_1(t)),\ X_2 < \Phi^{-1}(F_2(t)) \ldots \mid M)$$

$$= \prod_{m=1}^{n} \mathbb{P}(X_m < \Phi^{-1}(F_m(t)) \mid M)$$

$$= \prod_{m=1}^{n} \mathbb{P}(\sqrt{\rho}\,M + \sqrt{1-\rho}\,Z_m < \Phi^{-1}(F_m(t)) \mid M)$$

$$= \prod_{m=1}^{n} \mathbb{P}\left(Z_m < \frac{\Phi^{-1}(F_m(t)) - \sqrt{\rho}\,M}{\sqrt{1-\rho}} \,\middle|\, M\right)$$

$$= \prod_{m=1}^{n} \Phi\left(\frac{\Phi^{-1}(F_m(t)) - \sqrt{\rho}\,M}{\sqrt{1-\rho}}\right)$$

$$\mathbb{P}(\tau_1 < t, \ldots, \tau_n < t)$$

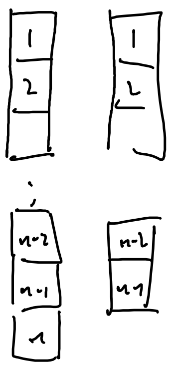
$$= \int_{-\infty}^{\infty} \prod_{m=1}^{n} \Phi\left(\frac{\Phi^{-1}(F_m(t)) - \sqrt{\rho}\, y}{\sqrt{1-\rho}} \right) e^{-\frac{1}{2}y^2} \frac{dy}{\sqrt{2\pi}}$$

$$\mathbb{P}(L(T) = m)$$

$L^{(n)}(T)$ = # of losses by T from first n entities

marginals equal "homogeneous" basket.

$$\mathbb{P}(L^{(n)}(T) = m \mid M)$$



m-1 def + "n" def OR m def + "n" no def

$$\mathbb{P}(L^{(n)}(T) = m \mid M) = \mathbb{P}(L^{(n-1)}(T) = m-1 \mid M) \cdot \mathbb{P}(X < \Phi^{-1}(F(T)) \mid M)$$

$$+ \mathbb{P}(L^{(n-1)}(T) = m \mid M) \cdot \left(1 - \mathbb{P}(X < \Phi^{-1}(F(T)) \mid M)\right)$$