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10:03 AM

CDS & CDO x 2

Commodity Models x 2

Adv int. rate x 2

Stochastic Vol. x 2

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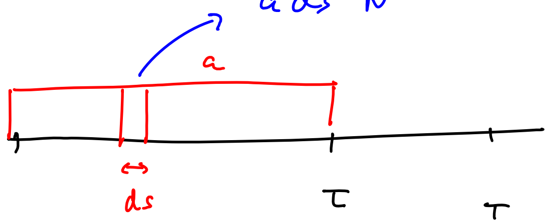
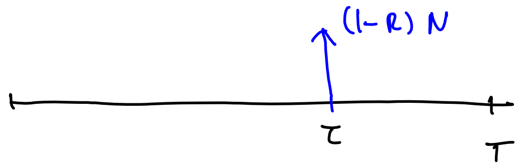
λ = const.

r = const.





" CDS = $(1-R)\lambda$ "

$$V_0^P = \int_0^T Q(\tau > s)\, e^{-rs}\, a N\, ds$$

$$= \int_0^T e^{-\lambda s} e^{-rs}\, a N\, ds$$

$$= a N \frac{1 - e^{-(\lambda+r)T}}{\lambda + r}$$

$$V_0^D = \int_0^T (1-R) N e^{-rs} \cdot \underbrace{Q(\tau \in (s, s+ds))}_{\lambda e^{-\lambda s} ds}$$

$$= (1-R) N \lambda \frac{1 - e^{-(\lambda+r)T}}{\lambda + r}$$

$$\Rightarrow \quad \boxed{a = (1-R)\lambda}$$

— $\lambda(s)$, $r(s)$ deterministic

$$V_0^P = \int_0^T \mathbb{Q}(\tau > s)\ a N\ e^{-\int_0^s r(u)\,du}\ ds$$

$$= \int_0^T e^{-\int_0^s \lambda(u)\,du}\ a N\ e^{-\int_0^s r(u)\,du}\ ds$$

$$= \int_0^T \exp\left\{-\int_0^s (\lambda(u) + r(u))\,du\right\} ds \quad a N$$

$$V_0^D = \int_0^T (1-R) N\ e^{-\int_0^s r(u)\,du} \cdot \lambda(s)\ e^{-\int_0^s \lambda(u)\,du}\ ds$$

$$= (1-R) N \int_0^T \exp\left\{-\int_0^s (\lambda(u) + r(u))\,du\right\} \lambda(s)\,ds$$

$$a = (1-R) \frac{\int_0^T e^{-\int_0^s (\lambda(u) + r(u))\,du}\ \lambda(s)\,ds}{\int_0^T e^{-\int_0^s (\lambda(u)+r(u))\,du}\ ds} \quad //$$

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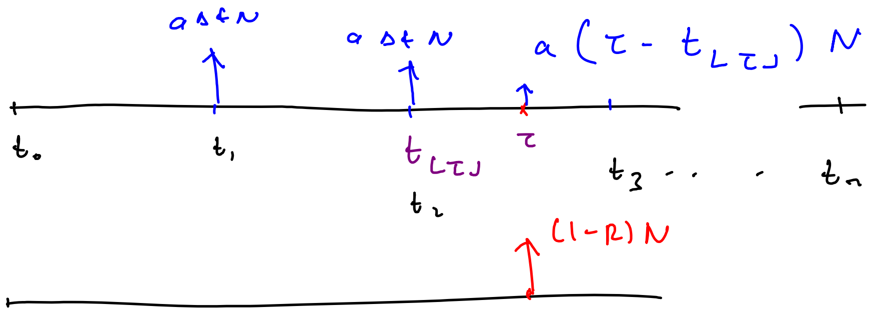
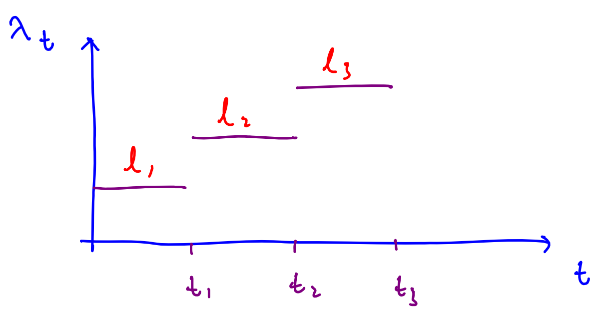


$$V_0^P = aN \sum_{i=1}^{n} \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^{t_i} r_s\, ds} \cdot \mathbb{1}_{\tau > t_i} \right]$$

$$+ aN \sum_{i=1}^{n} \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^{\tau} r_s\, ds} (\tau - t_{i-1}) \mathbb{1}_{\tau \in (t_{i-1}, t_i]} \right]$$



$$V_0^P = aN \sum_{i=1}^{n} P_0(t_i)\, e^{-\int_0^{t_i} \lambda_s\, ds}$$

$$+ aN \sum_{i=1}^{n} P_0(t_i) \left(\mathbb{E}^{\mathbb{Q}}\left[\tau\, \mathbb{1}_{\tau \in (t_{i-1}, t_i]} \right] - t_{i-1} \left(e^{-\int_0^{t_{i-1}} \lambda_s\, ds} - e^{-\int_0^{t_i} \lambda_s\, ds} \right) \right)$$

$\underline{i=1}$: $\mathbb{E}^{\mathbb{Q}}\left[\tau\, \mathbb{1}_{\tau \in (t_0, t_1]} \right]$

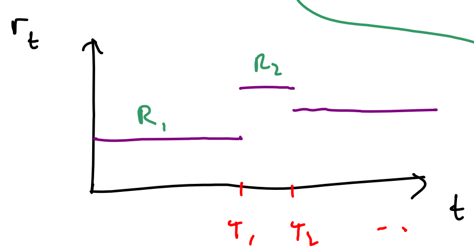
$$= \int_0^{t_1} s\, l_1\, e^{-l_1 s}\, ds = \ldots$$

$$\mathbb{E}^{Q}\left[\tau \, \mathbb{1}_{\tau \in (t_{i-1}, t_i]}\right]$$

$$= \int_{t_{i-1}}^{t_i} s \; \underbrace{l_i \cdot \exp\left\{ -\sum_{j=1}^{i-1} l_j (t_j - t_{j-1}) - l_i (s - t_{i-1}) \right\}}_{\lambda(s)\, e^{-\int_0^s \lambda(u)\,du}} \, ds$$

$$= \dots$$

$$V_0^d = \int_0^{t_n} (1-R) N \cdot P_0(s) \cdot \underbrace{\lambda(s)\, e^{-\int_0^s \lambda(u)\,du}} \, ds$$



$$\exp\left\{ -\sum R_i (t_i - t_{i-1}) - R_i (s - t_i) \right\}$$

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Doubly Stochastic Poisson Processes.

$$\mathbb{P}(N_{t+dt} - N_t = 1 \mid \mathcal{F}_t) = \lambda_t \, dt + o(dt)$$

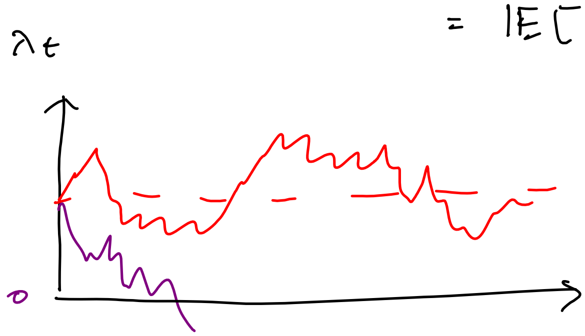
$$\mathbb{P}(N_{t+dt} - N_t = 0 \mid \mathcal{F}_t) = 1 - \lambda_t \, dt + o(dt)$$

$$\mathbb{P}(N_{t+dt} - N_t > 1 \mid \mathcal{F}_t) = o(dt).$$

$$\mathbb{P}(N_T = 0) = \mathbb{E}[\mathbb{1}_{\tau > T}]$$

$$= \mathbb{E}\Big[\underbrace{\mathbb{E}[\mathbb{1}_{\tau > T} \mid \sigma((\lambda_s)_{0 \le s \le T})]}_{e^{-\int_0^T \lambda_s \, ds}}\Big]$$

$$= \mathbb{E}[e^{-\int_0^T \lambda_s \, ds}]$$



$$d\lambda_t = \kappa(\theta - \lambda_t)\,dt + \sigma \, dW_t \qquad a)$$

$$d\lambda_t = \kappa(\theta - \lambda_t)\,dt + \sigma \sqrt{\lambda_t} \, dW_t \qquad b)$$

can make stochastic

How to compute $\mathbb{P}(\tau > T)$?

1. explicitly solve SDE
2. use distributional properties

$$g_t = \mathbb{E}_t\left[e^{-\int_0^T \lambda_s ds} \right] \quad \text{is a martingale!}$$

$$= \mathbb{E}\left[e^{-\int_0^T \lambda_s ds} \mid \mathcal{F}_t^\lambda \right]$$

$$\begin{cases} (\partial_t + \mathcal{L})\, g = 0 & g(t, \lambda, \Lambda) \\ \mathcal{L} = \kappa(\theta - \lambda)\partial_\lambda + \frac{1}{2}\sigma^2 \lambda \partial_\lambda + \lambda \partial_\Lambda & \end{cases}$$

$$g(T, \lambda, \Lambda) = \cancel{1} \to e^{-\Lambda}$$

$$g_t = g(t, \lambda_t, \underbrace{\int_0^t \lambda_s ds}_{= \Lambda_t})$$

$$d\Lambda_t = \lambda_t\, dt$$

$$d\lambda_t = \kappa(\theta - \lambda_t)dt + \sigma\sqrt{\lambda_t}\, dW_t$$

affine assumption:

$$g(t, \lambda, \Lambda) = \exp\{ A_t - B_t \lambda - C_t \Lambda \}$$

$$A_T = 0, \quad B_T = 0, \quad C_T = 1$$

$$\Rightarrow \quad g(t, \lambda, \Lambda) = e^{A_t - B_t \lambda - \Lambda}$$

$$g(0, \lambda_0, \underbrace{\Lambda_0}_{= \int_0^t \lambda_s ds}) = e^{A_0 - B_0 \lambda}$$